Masses of the pseudo-Nambu-Goldstone bosons in two flavor color superconducting phase

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The masses of the pseudo-Nambu-Goldstone bosons in the color superconducting phase of dense QCD with two light flavors are estimated by making use of the Cornwall-Jackiw-Tomboulis effective action. Parametrically, the masses of the doublet and antidoublet bosons are suppressed by a power of the coupling constant as compared to the value of the superconducting gap. This is qualitatively different from the mass expression for the singlet pseudo-Nambu-Goldstone boson, resulting from non-perturbative effects. It is argued that the (anti-) doublet pseudo-Nambu-Goldstone bosons form colorless [with respect to the unbroken $SU(2)_c$] charmonium-like bound states. The corresponding binding energy is also estimated.

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I. INTRODUCTION

Although there is no reliable observational signature yet, from theoretical point of view, it is quite reasonable to assume that the cores of compact stars are made of color superconducting quark matter [1]. If we take this assumption seriously, it becomes quite important to study the properties of the possible color superconducting phases in full detail (for an up to date review on color superconductivity see Refs. [2,3]). In this paper, we continue our study [4,5] of the pseudo-NG bosons, related to the approximate axial color symmetry, in the $S2C$ phase of cold dense QCD.

Let us recall that the axial color transformation is not a symmetry of the QCD action. However, its explicit breaking is a weak effect at sufficiently high quark densities where the coupling constant $\alpha_s(\mu)$ is small (μ is a chemical potential). This was our main argument in Refs. [4,5] suggesting the existence of five (rather than one [6]) light pseudo-NG bosons in the $S2C$ phase of cold dense QCD. No mass estimates for these pseudo-NG bosons were given in Refs. [4,5]. In this paper, we fill in the gap by developing the formalism and calculating the masses.

This paper is organized as follows. In the next section, we briefly introduce our model and notation. In Sec. III, we describe our method, based on the Cornwall-Jackiw-Tomboulis (CJT) effective action, for calculating mass estimates of the pseudo-NG bosons. Then, in Sec. IV, the leading order diagram is approximately calculated, using analytical methods. The fate of the colored pseudo-NG bosons in the doublet and antidoublet channels is discussed in Sec. V. In Sec. VI, we give our conclusions. In Appendices A and B we present the general expression for the gluon polarization tensor and the calculation of the integrals that appear in its definition. In Appendix C the problem of the gauge invariance in the loop expansion of the CJT effective action is discussed.

II. THE MODEL AND NOTATION

Here we consider cold dense QCD with two light quark flavors (u and d) in the fundamental representation of the $SU(3)_c$ color gauge group. In order to keep the analytical calculation under control, we assume that the chemical potential μ is much larger than Λ_{QCD}. Of course, when we talk about the compact stars, this is a far stretched assumption. Therefore, while extending our analytical results to the realistic densities existing, for example, at the cores of compact stars, one should be very careful. While most of the qualitative results may survive without being affected, most of the quantitative estimates would probably be valid only up to an order of magnitude. From the viewpoint of a theorist, it is still most interesting to study the predictions of the microscopic theory, *i.e.* QCD in the problem at hand. The price for such a luxury is the necessity to work at asymptotically large densities.

Instead of working with the standard four component Dirac spinors, in our analysis below, it is convenient to introduce the following eight component Majorana spinors:

$$\Psi = \frac{1}{\sqrt{2}} \begin{pmatrix} \psi_D \\ \psi_D^C \end{pmatrix}, \quad \psi_D^C = C\bar{\psi}_D^T, \tag{1}$$

where ψ_D is a Dirac spinor and C is a charge conjugation matrix, defined by $C^{-1}\gamma_\mu C = -\gamma_\mu^T$ and $C = -C^T$. In this notation, the inverse fermion propagator in the color superconducting phase reads [7–11]

$$\begin{aligned}(G(p))^{-1} &= -i \begin{pmatrix} (p_0+\mu)\gamma^0 + \vec{p}\!\!\!/ & \Delta \\ \tilde{\Delta} & (p_0-\mu)\gamma^0 + \vec{p}\!\!\!/ \end{pmatrix} \\ &= -i \begin{pmatrix} \gamma^0\left[(p_0-\epsilon_p^-)\Lambda_p^+ + (p_0+\epsilon_p^+)\Lambda_p^-\right] & \Delta \\ \tilde{\Delta} & \gamma^0\left[(p_0-\epsilon_p^+)\Lambda_p^+ + (p_0+\epsilon_p^-)\Lambda_p^-\right] \end{pmatrix},\end{aligned} \tag{2}$$

where $\tilde{\Delta} = \gamma^0\Delta^\dagger\gamma^0$, $\Delta_{ab}^{ij} \equiv \gamma^5\varepsilon^{ij}\varepsilon_{ab3}\left[\Delta^-\Lambda_p^- + \Delta^+\Lambda_p^+\right]$ and the "on-shell" projectors $\Lambda_p^{(\pm)}$ are

$$\Lambda_p^{(\pm)} = \frac{1}{2}\left(1 \pm \frac{\vec{\alpha}\cdot\vec{p}}{|\vec{p}|}\right), \quad \text{where} \quad \vec{\alpha} = \gamma^0\vec{\gamma}, \tag{3}$$

(note that $\Delta^\pm$ are complex valued gap functions). Color and flavor indices are denoted by small latin letters from the beginning and the middle of the alphabet, respectively. By definition, $\epsilon_p^\pm = |\vec{p}| \pm \mu$ and $\vec{p}\!\!\!/ = -\vec{p}\cdot\vec{\gamma}$. The effects of the quark wave function renormalization are neglected here.

Now, after inverting the expression in Eq. (2), we arrive at the following propagator:

$$G(p) = i \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix}, \tag{4}$$

where

$$R_{11}(p) = \gamma^0\mathcal{I}_1\left[\frac{p_0+\epsilon_p^-}{E_\Delta^-}\Lambda_p^- + \frac{p_0-\epsilon_p^+}{E_\Delta^+}\Lambda_p^+\right] + \gamma^0\mathcal{I}_2\left[\frac{1}{p_0-\epsilon_p^-}\Lambda_p^- + \frac{1}{p_0+\epsilon_p^+}\Lambda_p^+\right], \tag{5a}$$

$$R_{22}(p) = \gamma^0\mathcal{I}_1\left[\frac{p_0+\epsilon_p^+}{E_\Delta^+}\Lambda_p^- + \frac{p_0-\epsilon_p^-}{E_\Delta^-}\Lambda_p^+\right] + \gamma^0\mathcal{I}_2\left[\frac{1}{p_0-\epsilon_p^+}\Lambda_p^- + \frac{1}{p_0+\epsilon_p^-}\Lambda_p^+\right], \tag{5b}$$

$$R_{12}(p) = \gamma^5\hat{\varepsilon}\left[\frac{\Delta^+\Lambda_p^-}{E_\Delta^+} + \frac{\Delta^-\Lambda_p^+}{E_\Delta^-}\right], \tag{5c}$$

$$R_{21}(p) = -\gamma^5\hat{\varepsilon}\left[\frac{(\Delta^-)^*\Lambda_p^-}{E_\Delta^-} + \frac{(\Delta^+)^*\Lambda_p^+}{E_\Delta^+}\right], \tag{5d}$$

with $E_\Delta^\pm = p_0^2 - (\epsilon_p^\pm)^2 - |\Delta^\pm|^2$, and the three color-flavor matrices are defined as follows:

$$(\mathcal{I}_1)_{ab}^{ij} = (\delta_{ab} - \delta_{a3}\delta_{b3})\,\delta^{ij}, \tag{6}$$

$$(\mathcal{I}_2)_{ab}^{ij} = \delta_{a3}\delta_{b3}\delta^{ij}, \tag{7}$$

$$\hat{\varepsilon}_{ab}^{ij} = 2iT_{ab}^2\varepsilon^{ij}. \tag{8}$$

Notice that, when using this quark propagator in loop calculations, one should make the substitutions

$$E_\Delta^\pm \to E_\Delta^\pm + i\epsilon, \tag{9}$$

$$p_0 \pm \epsilon_p^- \to p_0 \pm \epsilon_p^- \mp i\epsilon\ \mathrm{sign}(\epsilon_p^-), \tag{10}$$

$$p_0 \pm \epsilon_p^+ \to p_0 \pm \epsilon_p^+ \mp i\epsilon, \tag{11}$$

in the denominators, and take the limit of vanishing ϵ at the end. This is important for preserving the causality of the theory.

III. DESCRIPTION OF THE FORMALISM

Let us start from a simple observation. If the model under consideration had real NG bosons in the spectrum, its effective potential as a function of the order parameter Δ_{ab}^{ij} would have a degenerate manifold of minima. The dimension of such a manifold would be equal to the number of the NG bosons. We know, however, that no global symmetries are broken in the $S2C$ phase and, therefore, the potential should have a single non-degenerate global minimum. The existence of the pseudo-NG bosons means, however, that the potential is nearly degenerate along selected directions. The curvature along these directions defines the masses of the pseudo-NG bosons. In the limit of the zero curvature, masses go to zero, as it should be for the NG bosons.

In order to select the directions in the color-flavor space that correspond to the five pseudo-NG bosons introduced in Ref. [4,5], we should recall their definition. These pseudo-NG bosons correspond to "breaking" of the approximate axial color symmetry, given by the following transformations of the quark fields:

$$\psi_D \to U\mathcal{P}_+\psi_D + U^\dagger\mathcal{P}_-\psi_D, \tag{12a}$$
$$\bar{\psi}_D \to \bar{\psi}_D\mathcal{P}_-U^\dagger + \bar{\psi}_D\mathcal{P}_+U, \tag{12b}$$
$$\psi_D^C \to U^*\mathcal{P}_-\psi_D^C + U^T\mathcal{P}_+\psi_D^C, \tag{12c}$$
$$\bar{\psi}_D^C \to \bar{\psi}_D^C\mathcal{P}_+U^T + \bar{\psi}_D^C\mathcal{P}_-U^*. \tag{12d}$$

Of course, this is *not* an exact symmetry of the model. For example, the kinetic term of the Lagrangian density transforms as follows:

$$\bar{\psi}_D\left(i\partial\!\!\!/ + \mu\gamma^0 + \hat{A}\!\!\!/\right)\psi_D \to \bar{\psi}_D\left(i\partial\!\!\!/ + \mu\gamma^0 + \mathcal{P}_+U\hat{A}\!\!\!/U^\dagger + \mathcal{P}_-U^\dagger\hat{A}\!\!\!/U\right)\psi_D, \tag{13}$$

and no transformation of the vector field could promote this transformation to a symmetry.

The axial color transformation, as defined above, allows us to explicitly extract the phase factors of the gap that correspond to the nearly degenerate directions of interest,

$$\Delta \to \mathcal{P}_+U^\dagger\Delta U^* + \mathcal{P}_-U\Delta U^T, \quad U = \exp\left(i\omega^A T^A\right). \tag{14}$$

One could consider ω^A as the dynamical fields of the pseudo-NG bosons (which, up to a factor of the decay constant, are related to the canonical fields). Such a substitution leads to the following changes of the components of the quark propagator:

$$R_{11} \to R_{11}(\omega) = \mathcal{P}_+U^\dagger R_{11}U + \mathcal{P}_-UR_{11}U^\dagger, \tag{15a}$$
$$R_{22} \to R_{22}(\omega) = \mathcal{P}_-U^T R_{22}U^* + \mathcal{P}_+U^*R_{22}U^T, \tag{15b}$$
$$R_{12} \to R_{12}(\omega) = \mathcal{P}_+U^\dagger R_{12}U^* + \mathcal{P}_-UR_{12}U^T, \tag{15c}$$
$$R_{21} \to R_{21}(\omega) = \mathcal{P}_-U^T R_{21}U + \mathcal{P}_+U^*R_{21}U^\dagger, \tag{15d}$$

assuming that the fields ω^A are constant in space-time.

In order to construct the effective potential, we use the CJT formalism [12]. The corresponding general expression reads

$$V = i\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left[\ln G(p)S^{-1}(p) - S^{-1}(p)G(p) + 1\right] + V_2[G], \tag{16}$$

where $V_2[G]$ represents the two-particle irreducible (with respect to quark lines) contributions (we will discuss this point below). There are, in general, an infinite number of diagrams in $V_2[G]$. In our analysis, we leave only a few leading order diagrams, graphically shown in Fig. 1.

Before proceeding to the actual calculation, let us try to understand which type of diagrams could produce a non-trivial dependence of the potential on the pseudo-NG fields ω^A. To this end, we have to recall the origin of the pseudo-NG bosons under consideration. In particular, it is crucial that their appearance is related to the breaking of the approximate axial color symmetry. It is clear, then, that the dependence of the effective potential on the pseudo-NG boson fields results from some mixing between the left-handed and the right-handed sectors of the theory. Since there is no any left-right mixing in the diagrams containing a single quark loop [diagrams (a), (b) and (c) in Fig. 1], the first three diagrams in Fig. 1 turn out to be irrelevant for our calculation. The corresponding contributions to the effective potential are free of any dependence on the constant ω^A fields.

Now, we move over to more complicated diagrams (d) and (e) in Fig. 1. Notice, that these last two diagrams are not two-particle irreducible with respect to gluon lines. This is consistent with the fact that we consider the CJT

action as a functional of only the quark propagator [for a discussion of this point see Ref. [13]]. Both diagrams could potentially produce non-trivial mass corrections for pseudo-NG bosons. In diagrams (d) and (e), a non-trivial mixing of the left- and right-handed quark sectors is possible because there are two separate quark loops.

A simple calculation shows that the diagram in Fig. 1d gives no correction to the effective potential. Thus, the leading order corrections come from the diagram in Fig. 1e. The details of our calculation are presented in the next section.

IV. LEADING ORDER CALCULATIONS

As we argued in the preceding section, the leading order corrections to the masses of the pseudo-NG bosons come from the diagram in Fig. 1e. In this section, we give the details of the calculation and derive an approximate analytical result for the masses.

The analytical expression for the vacuum diagram in Fig. 1e reads

$$V(\text{Fig. 1e}) = -2i\pi^2\alpha_s^2 \int \frac{d^4p d^4k d^4q}{(2\pi)^{12}} \mathrm{Tr}\left[\Gamma^{A\mu}G(p)\Gamma^{B\kappa}G(p-q)\right] \mathrm{Tr}\left[\Gamma^{A\nu}G(k-q)\Gamma^{B\lambda}G(k)\right] D_{\mu\nu}(q)D_{\kappa\lambda}(q), \tag{17}$$

where the vertex is

$$\Gamma^{A\mu} = \gamma^\mu \begin{pmatrix} T^A & 0 \\ 0 & -\left(T^A\right)^T \end{pmatrix}. \tag{18}$$

The expression in Eq. (17) contains two factors of the following type:

$$2i\pi\alpha_s \int \frac{d^4p}{(2\pi)^4} \mathrm{Tr}\left[\Gamma^{A\mu}G(p)\Gamma^{B\kappa}G(p-q)\right]. \tag{19}$$

In order to extract the dependence of this quantity on the psedo-NG boson fields ω^A, we perform the substitutions of all component functions, given in Eq. (15). At the end, we expand the result in powers of ω^A, keeping the terms up to the second order. Thus, we arrive at the following result:

$$\begin{aligned}
&2i\pi\alpha_s \int \frac{d^4p}{(2\pi)^4} \mathrm{tr}\Bigg[\mathcal{P}_+\gamma^\mu \tilde{T}^A_\omega R_{12}(p)\gamma^\kappa \left(\tilde{T}^B_\omega\right)^T R_{21}(p-q) + \mathcal{P}_-\gamma^\mu T^A_\omega R_{12}(p)\gamma^\kappa \left(T^B_\omega\right)^T R_{21}(p-q) \\
&+ \mathcal{P}_+\gamma^\mu \left(T^A_\omega\right)^T R_{21}(p)\gamma^\kappa T^B_\omega R_{12}(p-q) + \mathcal{P}_-\gamma^\mu \left(\tilde{T}^A_\omega\right)^T R_{21}(p)\gamma^\kappa \tilde{T}^B_\omega R_{12}(p-q) \\
&- \mathcal{P}_+\gamma^\mu \tilde{T}^A_\omega R_{11}(p)\gamma^\kappa \tilde{T}^B_\omega R_{11}(p-q) - \mathcal{P}_-\gamma^\mu T^A_\omega R_{11}(p)\gamma^\kappa T^B_\omega R_{11}(p-q) \\
&- \mathcal{P}_+\gamma^\mu \left(T^A_\omega\right)^T R_{22}(p)\gamma^\kappa \left(T^B_\omega\right)^T R_{22}(p-q) - \mathcal{P}_-\gamma^\mu \left(\tilde{T}^A_\omega\right)^T R_{22}(p)\gamma^\kappa \left(\tilde{T}^B_\omega\right)^T R_{22}(p-q)\Bigg] \\
&= \Pi^{AB,\mu\kappa}(q) + \omega^X\omega^Y f^{XAC} f^{YBD}\Pi^{CD,\mu\kappa}(q) \\
&+ \frac{1}{2}\omega^X\omega^Y f^{XAD} f^{YDC}\Pi^{CB,\mu\kappa}(q) + \frac{1}{2}\omega^X\omega^Y f^{XBD} f^{YDC}\Pi^{AC,\mu\kappa}(q).
\end{aligned} \tag{20}$$

Here we used the shorthand notation,

$$T^A_\omega \equiv U T^A U^\dagger \simeq T^A - \omega^B f^{BAC} T^C + \frac{1}{2}\omega^B\omega^C f^{BAD} f^{CDE} T^E + \dots, \tag{21a}$$

$$\tilde{T}^A_\omega \equiv U^\dagger T^A U \simeq T^A + \omega^B f^{BAC} T^C + \frac{1}{2}\omega^B\omega^C f^{BAD} f^{CDE} T^E + \dots, \tag{21b}$$

where f^{BAD} are the structure constants of $SU(3)$. Also, we introduced the one loop polarization tensor in the color superconducting phase [14],

$$\begin{aligned}
\Pi^{AB,\mu\kappa}(q) = 2i\pi\alpha_s \int \frac{d^4p}{(2\pi)^4} \mathrm{tr}\Bigg[&\gamma^\mu T^A R_{12}(p)\gamma^\kappa \left(T^B\right)^T R_{21}(p-q) + \gamma^\mu \left(T^A\right)^T R_{21}(p)\gamma^\kappa T^B R_{12}(p-q) \\
&- \gamma^\mu T^A R_{11}(p)\gamma^\kappa T^B R_{11}(p-q) - \gamma^\mu \left(T^A\right)^T R_{22}(p)\gamma^\kappa \left(T^B\right)^T R_{22}(p-q)\Bigg].
\end{aligned} \tag{22}$$

By performing the traces over the color and flavor indices, we arrive at the following expression for the polarization tensor (see Appendices A and B for details):

$$\Pi^{AB,\mu\nu}(q)\big|_{A,B=1,2,3} = \delta^{AB}\Pi_1^{\mu\nu}(q), \tag{23a}$$

$$\Pi^{AB,\mu\nu}(q)\big|_{A,B=4,5,6,7} = \delta^{AB}\Pi_4^{\mu\nu}(q) + i\left(\delta^{A4}\delta^{B5} - \delta^{A5}\delta^{B4} + \delta^{A6}\delta^{B7} - \delta^{A7}\delta^{B6}\right)\tilde{\Pi}_4^{\mu\kappa}(q), \tag{23b}$$

$$\Pi^{88,\mu\nu}(q) = \Pi_8^{\mu\nu}(q). \tag{23c}$$

After substituting the expansion (20) in Eq. (17), we arrive at

$$\begin{aligned} V(\text{Fig. 1e}) = {} & \frac{i}{2}\int \frac{d^4q}{(2\pi)^4}\Pi^{AB,\mu\kappa}(q)\Pi^{AB,\nu\lambda}(q)D_{\mu\nu}(q)D_{\kappa\lambda}(q) \\ & - \frac{3i}{4}\sum_{A=4}^{7}(\omega^A)^2\int \frac{d^4q}{(2\pi)^4}D_{\mu\nu}(q)D_{\kappa\lambda}(q) \\ & \times \Bigg\{ \left[\Pi_4^{\mu\kappa}(q) - \Pi_1^{\mu\kappa}(q)\right]\left[\Pi_4^{\nu\lambda}(q) - \Pi_1^{\nu\lambda}(q)\right] + 2\tilde{\Pi}_4^{\mu\kappa}(q)\tilde{\Pi}_4^{\nu\lambda}(q) \\ & + \left[\Pi_4^{\mu\kappa}(q) - \Pi_8^{\mu\kappa}(q)\right]\left[\Pi_4^{\nu\lambda}(q) - \Pi_8^{\nu\lambda}(q)\right] \Bigg\} + O\left[(\omega^A)^4\right]. \end{aligned} \tag{24}$$

It is noticeable that the right hand side of the last expression is independent of the ω^8 field, related to the singlet pseudo-NG boson. This means that the diagram in Fig. 1e does not give any non-trivial contribution to the value of the corresponding mass. Only the (anti-)doublet pseudo-NG bosons get a non-zero mass.

In order to understand this point, it is instructive to consider the "ideal" case, assuming that the axial color symmetry generated by $\gamma^5 T^8$ is a true (rather than approximate) symmetry of dense QCD. Then, the singlet pseudo-NG boson would be related to the breaking of the restricted axial symmetry, defined by the following transformation:

$$\psi \to \exp\left(i\omega\gamma^5\right)\mathcal{I}_1\psi + \mathcal{I}_2\psi, \tag{25a}$$

$$\bar{\psi} \to \bar{\psi}\mathcal{I}_1\exp\left(i\omega\gamma^5\right) + \bar{\psi}\mathcal{I}_2, \tag{25b}$$

$$\psi^C \to \exp\left(i\omega\gamma^5\right)\mathcal{I}_1\psi^C + \mathcal{I}_2\psi^C, \tag{25c}$$

$$\bar{\psi^C} \to \bar{\psi^C}\mathcal{I}_1\exp\left(i\omega\gamma^5\right) + \bar{\psi^C}\mathcal{I}_2. \tag{25d}$$

acting on the first two color quarks. The other symmetry $[\tilde{U}_5(1)]$, acting on the third color quarks would remain unbroken. This is a simple consequence of the fact that the third color quarks do not participate in the color condensation. Now, the axial color transformation generated by $\gamma^5 T^8$ could be thought of as the ordinary axial transformation $U_5(1)$ accompanied by the unbroken $\tilde{U}_5(1)$. Therefore, it appears to be equivalent to say that either the restricted or the ordinary axial symmetry is spontaneously broken in this ideal limit. In reality, both the restricted and the ordinary axial symmetries are explicitly broken. However, while the former is broken perturbatively, the latter is broken by much smaller non-perturbative (instanton-like) effects. Because of that, the singlet pseudo-NG boson is connected with the ordinary axial transformation $U_5(1)$ and its mass is zero in any order of the expansion in the coupling constant.

Non-perturbative analysis reveals that the singlet pseudo-NG boson η has a non-zero mass. The value of the mass was estimated in Ref. [15]. In our notation, it reads

$$M_\eta^2 \simeq \frac{C_\eta}{\alpha_s^7}|\Delta^-|^2\exp\left(-\frac{2\pi}{\alpha_s}\right), \qquad C_\eta \simeq 10^5. \tag{26}$$

The parametric dependence of this mass of the η pseudo-NG singlet clearly indicates the non-perturbative nature of the underlying dynamics. To derive this estimate, one needs to consider the instanton contribution (notice the characteristic exponential factor in the expression above) to the vacuum energy in the color superconducting phase.

Before calculating the masses of the other pseudo-NG bosons, let us turn to the question of the reliability (and, in particular, gauge invariance) of the loop expansion for the CJT effective potential in the present problem. From Eq. (24) we see that the vacuum energy is expressed through the one-loop polarization tensor. As is well known, the Ward (Slavnov-Taylor) identities (or, equivalently, the BRST transformations) imply that in both Abelian and non-Abelian gauge field theories, the polarization tensor is transverse in covariant gauges. We show in Appendix C,

however, that while the one-loop polarization tensor connected with the unbroken $SU(2)_c$ is indeed transverse [see Eqs. (C3), (C5) and (C8)], the one-loop polarization tensors connected with the five broken generators are not [see Eqs. (C4), (C6) and (C7) in Appendix C].

What is the reason for the violation of the Ward identities in the one-loop approximation? The answer is that it is connected with the Meissner-Higgs effect. As was already emphasized in our paper [5] and in the earlier papers [16,17], this effect implies the presence of unphysical composites having quantum numbers of the (would be) NG bosons in any non-unitary gauge, including of course all covariant ones. Although in the unitary gauge these composites are "eaten" by the five massive gluons, in other gauges they are crucial for preserving both the unitarity and the Ward identities (i.e. gauge invariance).[1] In particular, they lead to an important pole correction in the quark-gluon vertex function [5].

Actually, the traces of this problem are known to appear in other studies in dense QCD. For example, it shows up even in the analysis of the Schwinger-Dyson equation in the hard dense loop (HDL) improved ladder approximation [7,18]. In this case, the gauge dependence of the gap appears only through a dependence in the preexponential factor of the gap. Formally, it is the next-to-next-to-leading order correction. Usually, therefore, this gauge dependence of the superconducting gap is ignored without even studying its real origin. While such an attitude is quite harmless in the case of the Schwinger-Dyson equation, it turns out to be crucial in the present problem of calculating the masses of the pseudo-NG bosons. Indeed, as is shown in Appendix C, the gauge dependence of their masses, caused by the non-transversality of the polarization tensor, is strong. The reason is that while in the Schwinger-Dyson equation the dominant region is that with momenta much larger than the fermion gap $|\Delta^-|$, in this problem the infrared region with momenta less than $|\Delta^-|$ dominates.

This implies that a consistent approximation for calculating the CJT potential has to include those (would-be) NG composites. This in turn implies that one should modify the ordinary loop expansion of the CJT potential. Since here we are interested in the infrared dynamics, one can neglect the composite structure of these bosons. This leads to a non-linear realization of color symmetry breaking. In particular, one has to calculate the polarization tensor in this framework.

In Appendix C, we consider this problem. As is shown there, including the (would be) NG bosons indeed restores the transversality of the polarization tensor. The remarkable thing is that the dominant contribution to the masses of the pseudo-NG bosons comes only from the part connected with the magnetic gluons [singled out by the projection operator $O^{(1)}$, see Eq. (C1)], and this part, unlike the other contributions, remains unchanged by including the contribution of the would be NG composites. This implies that for the calculation of the masses of pseudo-NG bosons in the Landau gauge one can use the initial, unmodified framework for the CJT effective potential (at least in this approximation). Indeed, the dangerous longitudinal terms in the polarization operator does not contribute in this gauge and only the contribution of the magnetic gluons matters. Because of this observation, henceforth we will use this gauge.

To get a rough estimate of the mass of doublets, one could use the following order of magnitude asymptotes for the polarization tensor [see Eqs. (B8), (B14), (B20) and (B24) in Appendix B and Eq. (C1) in Appendix C]:

$$\Pi_4^{(1)}(q) - \Pi_1^{(1)}(q) \sim \Pi_4^{(1)}(q) - \Pi_8^{(1)}(q) \sim \begin{cases} \alpha_s\mu^2, & \text{for } |q_4|, q \ll 2|\Delta^-|; \\ \frac{\alpha_s\mu^2|\Delta^-|^2}{q_4^2}\ln\frac{q_4^2}{|\Delta^-|^2}, & \text{for } q, |\Delta^-| \ll |q_4|; \\ \frac{\alpha_s\mu^2|\Delta^-|^2}{|q_4|q}, & \text{for } |\Delta^-| \ll |q_4| \ll q; \\ \frac{\alpha_s\mu^2|\Delta^-|}{q}, & \text{for } |q_4| \ll |\Delta^-| \ll q. \end{cases} \tag{27}$$

By making use of these asymptotes along with the HDL expression for the gluon propagator [see, for example, Ref. [7]], we check that the dominant contribution to the quadratic term in Eq. (24) in the Landau gauge comes from the region of momenta where $|\Delta| \lesssim q \lesssim \mu$ and $0 \lesssim |q_4| \lesssim |\Delta|$. Therefore, by taking the explicit expressions for the polarization tensor given in Appendix C into account, we derive

$$\begin{aligned} M^2 = \frac{1}{F^2}\frac{\partial^2 V}{\partial\omega^2} &\simeq \frac{170\pi}{3\mu^2}\int_{|\Delta|}^{\mu} q^2 dq \int_0^{|\Delta|} dq_4 \frac{q^2}{\left(q^3 + \frac{\pi}{2}m_D^2|q_4|\right)^2}\left(\frac{\alpha_s\mu^2|\Delta|}{q}\right)^2 \\ &\simeq \frac{340\pi}{9}\alpha_s|\Delta|^2 \ln\frac{\mu}{|\Delta|} \simeq C_M\sqrt{\alpha_s}|\Delta|^2, \quad C_M \approx \frac{85\pi^2}{3}\sqrt{2\pi} \approx 7\times 10^2, \end{aligned} \tag{28}$$

[1]Note that, because of the composite (diquark) nature of the order parameter in color superconductivity, it does not seem to be straightforward to define and to use the unitary gauge in this case.

where the definition of the Debye mass, $m_D^2 = (2/\pi)\alpha_s \mu^2$, the decay constant $F^2 = \mu^2/8\pi^2$, as well as the relation between the gap and the chemical potential, $\ln(\mu/|\Delta|) \simeq 3(\pi/2)^{3/2}\alpha_s^{-1/2}$, were used.

V. PSEUDO-NG (ANTI-) DOUBLETS VERSUS CONFINEMENT

In the color superconducting phase in the model with two quark flavors, the $SU(2)_c$ subgroup remains unbroken. Since the only massless quarks of the third color do not interact with gluons of $SU(2)_c$, the corresponding dynamics of gluons decouples in the far infrared region, $p \ll |\Delta^-|$. The low energy effective action of the $SU(2)_c$ gluondynamics was derived in Ref. [19]. Of special interest for us here is the observation of Ref. [19] that the unbroken $SU(2)_c$ is confined at sufficiently low energies. The corresponding scale was also estimated. It is given by the following simple expression:

$$\Lambda'_{QCD} \simeq |\Delta^-| \exp\left[-C_0 \alpha_s^2 \exp\left(C/\sqrt{\alpha_s}\right)\right], \qquad C = 3\left(\frac{\pi}{2}\right)^{3/2}, \tag{29}$$

(there is some uncertainty in determining $C_0 \simeq 10^{-3}$).

Now, let us discuss how such confinement could affect the properties of the pseudo-NG bosons. In particular, this concerns the doublet and antidoublet states which are colored under the leftover $SU(2)_c$ subgroup. Because of the confinement, these pseudo-NG states cannot exist as free particles. Instead, they should form some colorless bound states.

The colorless bound states should be somewhat similar to charmonium in ordinary QCD. In fact, the situation with the doublet-antidoublet states in the color superconducting phase is much simpler. This is because the new confinement scale Λ'_{QCD} is extremely small as compared to the masses of the (anti-) doublets themselves. This already suggests that the binding dynamics is essentially perturbative [this could also be checked posterior, see Eq. (32) below]. The bound colorless states would be similar to the positronium in QED [20–22]. The only difference is due to the color superconducting medium effects. The Coulomb potential between two static charges at a distance r from each other is given by [19]

$$V_C \simeq \frac{4\pi\alpha_s}{\epsilon r}, \qquad \frac{1}{|\Delta^-|} \ll r \ll \frac{1}{\Lambda'_{QCD}}, \tag{30}$$

where

$$\epsilon \approx \frac{2\alpha_s \mu^2}{9\pi|\Delta^-|^2} \tag{31}$$

is the dielectric constant [19] [see also our derivation of Eq. (C3) in Appendix C]. By analogy with the positronium (notice that the constituent doublets are spinless in the problem at hand, but such a difference affects only the fine structure of the spectrum), we get the following estimate for the binding energies of the colorless states built of the doublet-antidoublet pairs:

$$E_n \simeq -\frac{M\alpha_s^2}{4\epsilon^2 n^2} \simeq C_E \frac{\alpha_s^{-39/4}|\Delta^-|}{n^2} \exp\left(-\frac{3\sqrt{2}\pi^{3/2}}{\sqrt{\alpha_s}}\right), \qquad n = 1, 2, \ldots, \tag{32}$$

with $C_E \approx 1.8 \times 10^{11}$. Notice that this binding energy is parametrically much larger than the confinement scale Λ'_{QCD} in Eq. (29) when $\alpha_s \to 0$. This means that it is the (perturbative) Coulomb contribution to the potential that is mostly responsible for the paring dynamics of the (anti-) doublets into colorless hadrons. The linear confining (non-perturbative) part of the potential is of minor importance at least for the low lying states.

We recall that in the $S2C$ phase, there exists a conserved (modified) baryon charge. While the value of this charge equals zero for the quarks of the first two colors, it is $+1$ for the third color (massless) quarks. Since the pseudo-NG (anti-) doublets are composed of one massive (anti-) quark and one massless (anti-) quark, they are scalar baryons, carrying the baryon charge ± 1. The colorless hadrons composed of them (discussed above) can be called mesons.

VI. CONCLUSION

In this paper, we derived analytical estimates for the masses of the pseudo-Nambu-Goldstone bosons in the color superconducting phase of dense QCD with two light flavors. In agreement with our previous hypothesis, the mass

values are small compared to the value of the superconducting gap at sufficiently large quark densities. Analytically, the expression for the masses of doublet and antidoublet pseudo-NG bosons are suppressed by a power of the coupling constant. The mass of the singlet is exponentially suppressed. The mechanism for mass generation of the singlet is purely non-perturbative [15].

As was shown in [19], the unbroken $SU(2)_c$ subgroup is subject to confinement. This fact has an important effect on the properties of the doublet and antidoublet pseudo-NG bosons which are colored with respect to $SU(2)_c$. In particular, we argue that the doublets and antidoublets should form charmonium-like colorless bound states. Moreover, since the confinement scale of $SU(2)_c$ subgroup is extremely small compared to the value of the masses of doublet and antidoublet, the pairing dynamics of the colorless mesons is dominated by perturbative Coulomb-like forces. This allows us to estimate the corresponding binding energy.

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APPENDIX A: GENERAL FORM OF THE POLARIZATION TENSOR

The general form of the polarization tensor in the color superconducting phase was given in [14]. For completeness of the presentation, we also rederive it here

$$\begin{aligned}
&\Pi^{AB,\mu\nu}(q)\big|_{A,B=1,2,3} \equiv \delta^{AB}\Pi_1^{\mu\nu}(q) = 2i\delta^{AB}\pi\alpha_s \int \frac{d^4p}{(2\pi)^4} \\
&\times \mathrm{tr}_D\Bigg[\gamma^\mu\left(\frac{\Delta^+\Lambda_p^-}{E_\Delta^+(p)}+\frac{\Delta^-\Lambda_p^+}{E_\Delta^-(p)}\right)\gamma^\nu\left(\frac{(\Delta^-)^*\Lambda_{p-q}^-}{E_\Delta^-(p-q)}+\frac{(\Delta^+)^*\Lambda_{p-q}^+}{E_\Delta^+(p-q)}\right) \\
&+\gamma^\mu\left(\frac{(\Delta^-)^*\Lambda_p^-}{E_\Delta^-(p)}+\frac{(\Delta^+)^*\Lambda_p^+}{E_\Delta^+(p)}\right)\gamma^\nu\left(\frac{\Delta^+\Lambda_{p-q}^-}{E_\Delta^+(p-q)}+\frac{\Delta^-\Lambda_{p-q}^+}{E_\Delta^-(p-q)}\right) \\
&+\gamma^\mu\gamma^0\left(\frac{p_0+\epsilon_p^-}{E_\Delta^-(p)}\Lambda_p^-+\frac{p_0-\epsilon_p^+}{E_\Delta^+(p)}\Lambda_p^+\right)\gamma^\nu\gamma^0\left(\frac{p_0-q_0+\epsilon_{p-q}^-}{E_\Delta^-(p-q)}\Lambda_{p-q}^-+\frac{p_0-q_0-\epsilon_{p-q}^+}{E_\Delta^+(p-q)}\Lambda_{p-q}^+\right) \\
&+\gamma^\mu\gamma^0\left(\frac{p_0+\epsilon_p^+}{E_\Delta^+(p)}\Lambda_p^-+\frac{p_0-\epsilon_p^-}{E_\Delta^-(p)}\Lambda_p^+\right)\gamma^\nu\gamma^0\left(\frac{p_0-q_0+\epsilon_{p-q}^+}{E_\Delta^+(p-q)}\Lambda_{p-q}^-+\frac{p_0-q_0-\epsilon_{p-q}^-}{E_\Delta^-(p-q)}\Lambda_{p-q}^+\right)\Bigg],
\end{aligned}\tag{A1}$$

$$\begin{aligned}
&\Pi^{AB,\mu\nu}(q)\big|_{A,B=4,5,6,7} \equiv \delta^{AB}\Pi_4^{\mu\nu}(q) + i\left(\delta^{A4}\delta^{B5}-\delta^{A5}\delta^{B4}+\delta^{A6}\delta^{B7}-\delta^{A7}\delta^{B6}\right)\tilde{\Pi}_4^{\mu\nu}(q) = i\pi\alpha_s\delta^{AB} \\
&\times\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}_D\Bigg[\gamma^\mu\gamma^0\left(\frac{p_0+\epsilon_p^-}{E_\Delta^-(p)}\Lambda_p^-+\frac{p_0-\epsilon_p^+}{E_\Delta^+(p)}\Lambda_p^+\right)\gamma^\nu\gamma^0\left(\frac{1}{p_0-q_0-\epsilon_{p-q}^-}\Lambda_{p-q}^-+\frac{1}{p_0-q_0+\epsilon_{p-q}^+}\Lambda_{p-q}^+\right) \\
&+\gamma^\mu\gamma^0\left(\frac{1}{p_0-\epsilon_p^-}\Lambda_p^-+\frac{1}{p_0+\epsilon_p^+}\Lambda_p^+\right)\gamma^\nu\gamma^0\left(\frac{p_0-q_0+\epsilon_{p-q}^-}{E_\Delta^-(p-q)}\Lambda_{p-q}^-+\frac{p_0-q_0-\epsilon_{p-q}^+}{E_\Delta^+(p-q)}\Lambda_{p-q}^+\right) \\
&+\gamma^\mu\gamma^0\left(\frac{p_0+\epsilon_p^+}{E_\Delta^+(p)}\Lambda_p^-+\frac{p_0-\epsilon_p^-}{E_\Delta^-(p)}\Lambda_p^+\right)\gamma^\nu\gamma^0\left(\frac{1}{p_0-q_0-\epsilon_{p-q}^+}\Lambda_{p-q}^-+\frac{1}{p_0-q_0+\epsilon_{p-q}^-}\Lambda_{p-q}^+\right) \\
&+\gamma^\mu\gamma^0\left(\frac{1}{p_0-\epsilon_p^+}\Lambda_p^-+\frac{1}{p_0+\epsilon_p^-}\Lambda_p^+\right)\gamma^\nu\gamma^0\left(\frac{p_0-q_0+\epsilon_{p-q}^+}{E_\Delta^+(p-q)}\Lambda_{p-q}^-+\frac{p_0-q_0-\epsilon_{p-q}^-}{E_\Delta^-(p-q)}\Lambda_{p-q}^+\right)\Bigg] \\
&-\frac{\pi}{2}\alpha_s\left(\delta^{A4}\delta^{B5}-\delta^{A5}\delta^{B4}+\delta^{A6}\delta^{B7}-\delta^{A7}\delta^{B6}\right)
\end{aligned}$$

$$
\times \int \frac{d^4p}{(2\pi)^4} \mathrm{tr}_D \Bigg[\gamma^\mu\gamma^0 \left(\frac{p_0+\epsilon_p^-}{E_\Delta^-(p)}\Lambda_p^- + \frac{p_0-\epsilon_p^+}{E_\Delta^+(p)}\Lambda_p^+ \right) \gamma^\nu\gamma^0 \left(\frac{1}{p_0-q_0-\epsilon_{p-q}^-}\Lambda_{p-q}^- + \frac{1}{p_0-q_0+\epsilon_{p-q}^+}\Lambda_{p-q}^+ \right)
$$

$$
- \gamma^\mu\gamma^0 \left(\frac{1}{p_0-\epsilon_p^-}\Lambda_p^- + \frac{1}{p_0+\epsilon_p^+}\Lambda_p^+ \right) \gamma^\nu\gamma^0 \left(\frac{p_0-q_0+\epsilon_{p-q}^-}{E_\Delta^-(p-q)}\Lambda_{p-q}^- + \frac{p_0-q_0-\epsilon_{p-q}^+}{E_\Delta^+(p-q)}\Lambda_{p-q}^+ \right)
$$

$$
+ \gamma^\mu\gamma^0 \left(\frac{p_0+\epsilon_p^+}{E_\Delta^+(p)}\Lambda_p^- + \frac{p_0-\epsilon_p^-}{E_\Delta^-(p)}\Lambda_p^+ \right) \gamma^\nu\gamma^0 \left(\frac{1}{p_0-q_0-\epsilon_{p-q}^+}\Lambda_{p-q}^- + \frac{1}{p_0-q_0+\epsilon_{p-q}^-}\Lambda_{p-q}^+ \right)
$$

$$
- \gamma^\mu\gamma^0 \left(\frac{1}{p_0-\epsilon_p^+}\Lambda_p^- + \frac{1}{p_0+\epsilon_p^-}\Lambda_p^+ \right) \gamma^\nu\gamma^0 \left(\frac{p_0-q_0+\epsilon_{p-q}^+}{E_\Delta^+(p-q)}\Lambda_{p-q}^- + \frac{p_0-q_0-\epsilon_{p-q}^-}{E_\Delta^-(p-q)}\Lambda_{p-q}^+ \right) \Bigg], \tag{A2}
$$

$$
\Pi^{88,\mu\nu}(q) \equiv \Pi_8^{\mu\nu}(q) = -\frac{2}{3} i\pi\alpha_s \int \frac{d^4p}{(2\pi)^4}
$$

$$
\times \mathrm{tr}_D \Bigg[\gamma^\mu \left(\frac{\Delta^+\Lambda_p^-}{E_\Delta^+(p)} + \frac{\Delta^-\Lambda_p^+}{E_\Delta^-(p)} \right) \gamma^\nu \left(\frac{(\Delta^-)^*\Lambda_{p-q}^-}{E_\Delta^-(p-q)} + \frac{(\Delta^+)^*\Lambda_{p-q}^+}{E_\Delta^+(p-q)} \right)
$$

$$
+ \gamma^\mu \left(\frac{(\Delta^-)^*\Lambda_p^-}{E_\Delta^-(p)} + \frac{(\Delta^+)^*\Lambda_p^+}{E_\Delta^+(p)} \right) \gamma^\nu \left(\frac{\Delta^+\Lambda_{p-q}^-}{E_\Delta^+(p-q)} + \frac{\Delta^-\Lambda_{p-q}^+}{E_\Delta^-(p-q)} \right)
$$

$$
- \gamma^\mu\gamma^0 \left(\frac{p_0+\epsilon_p^-}{E_\Delta^-(p)}\Lambda_p^- + \frac{p_0-\epsilon_p^+}{E_\Delta^+(p)}\Lambda_p^+ \right) \gamma^\nu\gamma^0 \left(\frac{p_0-q_0+\epsilon_{p-q}^-}{E_\Delta^-(p-q)}\Lambda_{p-q}^- + \frac{p_0-q_0-\epsilon_{p-q}^+}{E_\Delta^+(p-q)}\Lambda_{p-q}^+ \right)
$$

$$
- \gamma^\mu\gamma^0 \left(\frac{p_0+\epsilon_p^+}{E_\Delta^+(p)}\Lambda_p^- + \frac{p_0-\epsilon_p^-}{E_\Delta^-(p)}\Lambda_p^+ \right) \gamma^\nu\gamma^0 \left(\frac{p_0-q_0+\epsilon_{p-q}^+}{E_\Delta^+(p-q)}\Lambda_{p-q}^- + \frac{p_0-q_0-\epsilon_{p-q}^-}{E_\Delta^-(p-q)}\Lambda_{p-q}^+ \right)
$$

$$
- 2\gamma^\mu\gamma^0 \left(\frac{1}{p_0-\epsilon_p^-}\Lambda_p^- + \frac{1}{p_0+\epsilon_p^+}\Lambda_p^+ \right) \gamma^\nu\gamma^0 \left(\frac{1}{p_0-q_0-\epsilon_{p-q}^-}\Lambda_{p-q}^- + \frac{1}{p_0-q_0+\epsilon_{p-q}^+}\Lambda_{p-q}^+ \right)
$$

$$
- 2\gamma^\mu\gamma^0 \left(\frac{1}{p_0-\epsilon_p^+}\Lambda_p^- + \frac{1}{p_0+\epsilon_p^-}\Lambda_p^+ \right) \gamma^\nu\gamma^0 \left(\frac{1}{p_0-q_0-\epsilon_{p-q}^+}\Lambda_{p-q}^- + \frac{1}{p_0-q_0+\epsilon_{p-q}^-}\Lambda_{p-q}^+ \right) \Bigg]. \tag{A3}
$$

Here we made use of the the following results for color-flavor traces:

$$
\mathrm{tr}_{cf}\left(\mathcal{I}_1 T^A \mathcal{I}_1 T^B\right) = \begin{cases} \delta^{AB} & A,B=1,2,3; \\ 0 & A,B=4,5,6,7; \\ \frac{1}{3} & A,B=8; \end{cases} \tag{A4}
$$

$$
\mathrm{tr}_{cf}\left(\mathcal{I}_2 T^A \mathcal{I}_2 T^B\right) = \begin{cases} 0 & A,B=1,\ldots,7; \\ \frac{2}{3} & A,B=8; \end{cases} \tag{A5}
$$

$$
\mathrm{tr}_{cf}\left(\mathcal{I}_1 T^A \mathcal{I}_2 T^B\right) = \begin{cases} 0 & A,B=1,2,3; \\ \frac{1}{2}\delta^{AB} + \frac{i}{4}\left(\delta^{A4}\delta^{B5} - \delta^{A5}\delta^{B4} + \delta^{A6}\delta^{B7} - \delta^{A7}\delta^{B6}\right) & A,B=4,5,6,7; \\ 0 & A,B=8; \end{cases} \tag{A6}
$$

$$
\mathrm{tr}_{cf}\left[\hat{\varepsilon} T^A \hat{\varepsilon} \left(T^B\right)^T\right] = \begin{cases} -\delta^{AB} & A,B=1,2,3; \\ 0 & A,B=4,5,6,7; \\ \frac{1}{3} & A,B=8. \end{cases} \tag{A7}
$$

APPENDIX B: CALCULATION OF INTEGRALS

In this Appendix, we calculate different type of integrals that appear in the expression for the polarization tensor (see the previous Appendix). For our purposes it is sufficient to consider the gluon momenta much less than μ, neglecting all corrections of order q^2/μ^2. In this approximation, the Dirac traces, containing the on-shell projection operators, read

$$\mathrm{tr}\left[\gamma^\mu \Lambda_p^{(\pm)} \gamma^\nu \Lambda_{p-q}^{(\pm)}\right] \simeq 2\left(g^{\mu\nu} - g^{\mu 0} g^{\nu 0} + \frac{\vec{p}^\mu \vec{p}^\nu}{|\vec{p}|^2}\right), \tag{B1}$$

$$\mathrm{tr}\left[\gamma^\mu \Lambda_p^{(\pm)} \gamma^\nu \Lambda_{p-q}^{(\mp)}\right] \simeq 2\left(g^{\mu 0} g^{\nu 0} - \frac{\vec{p}^\mu \vec{p}^\nu}{|\vec{p}|^2}\right), \tag{B2}$$

$$\mathrm{tr}\left[\gamma^\mu \gamma^0 \Lambda_p^{(\pm)} \gamma^\nu \gamma^0 \Lambda_{p-q}^{(\pm)}\right] \simeq 2\left(g^{\mu 0} \mp \frac{\vec{p}^\mu}{|\vec{p}|}\right)\left(g^{\nu 0} \mp \frac{\vec{p}^\nu}{|\vec{p}|}\right), \tag{B3}$$

$$\mathrm{tr}\left[\gamma^\mu \gamma^0 \Lambda_p^{(\pm)} \gamma^\nu \gamma^0 \Lambda_{p-q}^{(\mp)}\right] \simeq -2\left(g^{\mu\nu} - g^{\mu 0} g^{\nu 0} + \frac{\vec{p}^\mu \vec{p}^\nu}{|\vec{p}|^2}\right). \tag{B4}$$

Also notice that in order to perform the integrations properly, one should make the following replacements in the denominators of propagators:

$$E_\Delta^\pm \to E_\Delta^\pm + i\varepsilon, \tag{B5}$$

$$p_0 \pm \epsilon_p^- \to p_0 \pm \epsilon_p^- \mp i\varepsilon\ \mathrm{sign}(\epsilon_p^-), \tag{B6}$$

$$p_0 \pm \epsilon_p^+ \to p_0 \pm \epsilon_p^+ \mp i\varepsilon. \tag{B7}$$

Form the definition in the previous Appendix, we see that the general structure of the polarization tensor reads

$$\Pi_1^{\mu\nu}(q) = 4\pi\alpha_s \left(J_\Delta^{\mu\nu}(q) + I_\Delta^{\mu\nu}(q)\right), \tag{B8a}$$

$$\Pi_4^{\mu\nu}(q) = 4\pi\alpha_s \tilde{I}_\Delta^{\mu\nu}(q), \tag{B8b}$$

$$\Pi_8^{\mu\nu}(q) = -\frac{4\pi\alpha_s}{3}\left(J_\Delta^{\mu\nu}(q) - I_\Delta^{\mu\nu}(q)\right) + \frac{8\pi\alpha_s}{3} I_{HDL}^{\mu\nu}(q). \tag{B8c}$$

[Here we took into account the fact that the off-diagonal term in $\Pi^{AB,\mu\nu}(q)\big|_{A,B=4,5,6,7}$ is zero. To see this, one has to calculate the corresponding integral in Eq. (A2).] Let us start with the calculation of the first type of the integrals,

$$\begin{aligned}
I_{HDL}^{\mu\nu}(q) &= i\int \frac{d^4p}{(2\pi)^4} \mathrm{tr}_D \left[\gamma^\mu\gamma^0\left(\frac{1}{p_0-\epsilon_p^-}\Lambda_p^- + \frac{1}{p_0+\epsilon_p^+}\Lambda_p^+\right)\gamma^\nu\gamma^0\left(\frac{1}{p_0-q_0-\epsilon_{p-q}^-}\Lambda_{p-q}^- + \frac{1}{p_0-q_0+\epsilon_{p-q}^+}\Lambda_{p-q}^+\right)\right] \\
&= i\int \frac{d^4p}{(2\pi)^4} \mathrm{tr}_D \left[\gamma^\mu\gamma^0\left(\frac{1}{p_0-\epsilon_p^+}\Lambda_p^- + \frac{1}{p_0+\epsilon_p^-}\Lambda_p^+\right)\gamma^\nu\gamma^0\left(\frac{1}{p_0-q_0-\epsilon_{p-q}^+}\Lambda_{p-q}^- + \frac{1}{p_0-q_0+\epsilon_{p-q}^-}\Lambda_{p-q}^+\right)\right] \\
&= -2\int \frac{d^3p}{(2\pi)^3}\left(g^{\mu 0} + \frac{\vec{p}^\mu}{|\vec{p}|}\right)\left(g^{\nu 0} + \frac{\vec{p}^\nu}{|\vec{p}|}\right)\left(\frac{\theta(-\epsilon_p^-)\theta(\epsilon_{p-q}^-)}{\epsilon_p^- - \epsilon_{p-q}^- - q_0 + i\varepsilon} + \frac{\theta(\epsilon_p^-)\theta(-\epsilon_{p-q}^-)}{\epsilon_{p-q}^- - \epsilon_p^- + q_0 + i\varepsilon}\right) \\
&\quad -2\int \frac{d^3p}{(2\pi)^3}\left(g^{\mu\nu} - g^{\mu 0}g^{\nu 0} + \frac{\vec{p}^\mu\vec{p}^\nu}{|\vec{p}|^2}\right)\left(\frac{\theta(\epsilon_p^-)}{p} - \frac{1}{p}\right) \\
&= -\frac{\mu^2}{\pi^2}\Bigg[g^{\mu 0}g^{\nu 0} Q\left(\frac{q_0}{q}\right) - \frac{1}{2}\left(g^{\mu\nu} - g^{\mu 0}g^{\nu 0} + \frac{\vec{q}^\mu\vec{q}^\nu}{q^2}\right)\left(1 + \frac{q^2 - q_0^2}{q^2} Q\left(\frac{q_0}{q}\right)\right) \\
&\quad + \frac{\vec{q}^\mu\vec{q}^\nu}{q^2}\frac{q_0^2}{q^2} Q\left(\frac{q_0}{q}\right) + \frac{q_0}{q}\left(g^{\mu 0}\frac{\vec{q}^\nu}{q} + g^{\nu 0}\frac{\vec{q}^\mu}{q}\right) Q\left(\frac{q_0}{q}\right)\Bigg],
\end{aligned} \tag{B9}$$

where

$$Q(x) \equiv -\frac{1}{2}\int_0^1 d\xi \left(\frac{\xi}{\xi + x - i\varepsilon} + \frac{\xi}{\xi - x - i\varepsilon}\right) = \frac{x}{2}\ln\left|\frac{1+x}{1-x}\right| - 1 - i\frac{\pi}{2}|x|\theta(1-x^2). \tag{B10}$$

It is easy to check that $q_\mu I_{HDL}^{\mu\nu}(q) = 0$, as it should be. Also notice that in Eucledian space, $x = iq_4/q \equiv iy$, and

$$Q(x) \to \tilde{Q}(y) = -1 + y\arctan\frac{1}{y} \simeq \begin{cases} -1 + \frac{\pi}{2}y, & \text{for } y \ll 1; \\ -\frac{1}{3y^2}, & \text{for } y \gg 1. \end{cases} \tag{B11}$$

Now, let us consider

$$
\begin{aligned}
J^{\mu\nu}_{\Delta}(q) &= i\int \frac{d^4p}{(2\pi)^4}\mathrm{tr}_D\left[\gamma^\mu\left(\frac{\Delta^+\Lambda^-_p}{E^+_\Delta(p)}+\frac{\Delta^-\Lambda^+_p}{E^-_\Delta(p)}\right)\gamma^\nu\left(\frac{(\Delta^-)^*\Lambda^-_{p-q}}{E^-_\Delta(p-q)}+\frac{(\Delta^+)^*\Lambda^+_{p-q}}{E^+_\Delta(p-q)}\right)\right]\\
&= i\int \frac{d^4p}{(2\pi)^4}\mathrm{tr}_D\left[\gamma^\mu\left(\frac{(\Delta^-)^*\Lambda^-_p}{E^-_\Delta(p)}+\frac{(\Delta^+)^*\Lambda^+_p}{E^+_\Delta(p)}\right)\gamma^\nu\left(\frac{\Delta^+\Lambda^-_{p-q}}{E^+_\Delta(p-q)}+\frac{\Delta^-\Lambda^+_{p-q}}{E^-_\Delta(p-q)}\right)\right]\\
&\simeq 2i\int \frac{d^4p}{(2\pi)^4}\left(g^{\mu0}g^{\nu0}-\frac{\vec{p}^\mu\vec{p}^\nu}{p^2}\right)\frac{|\Delta^-|^2}{E^-_\Delta(p)E^-_\Delta(p-q)}\\
&= -\frac{\mu^2}{4\pi^2}\int_0^1 d\xi\int_0^1 dx\frac{|\Delta^-|^2\left[(1+\xi^2)g^{\mu0}g^{\nu0}+(1-\xi^2)g^{\mu\nu}+(1-3\xi^2)\frac{\vec{q}^\mu\vec{q}^\nu}{q^2}\right]}{|\Delta^-|^2+x(1-x)\left(\xi^2q^2-q_0^2\right)-i\varepsilon}.
\end{aligned}
\tag{B12}
$$

After switching to the Eucledian momenta ($q_0=iq_4$), we get

$$
\begin{aligned}
J^{\mu\nu}_{\Delta}(q) &= -\frac{\mu^2}{4\pi^2}\int_0^1 d\xi\int_0^1 dx\frac{|\Delta^-|^2\left[(1+\xi^2)g^{\mu0}g^{\nu0}+(1-\xi^2)g^{\mu\nu}+(1-3\xi^2)\frac{\vec{q}^\mu\vec{q}^\nu}{q^2}\right]}{|\Delta^-|^2+x(1-x)\left(q_4^2+\xi^2q^2\right)}\\
&= -\frac{\mu^2}{2\pi^2}\int_0^1 d\xi\frac{|\Delta^-|^2\left[(1+\xi^2)g^{\mu0}g^{\nu0}+(1-\xi^2)g^{\mu\nu}+(1-3\xi^2)\frac{\vec{q}^\mu\vec{q}^\nu}{q^2}\right]}{\sqrt{q_4^2+\xi^2q^2}\sqrt{q_4^2+\xi^2q^2+4|\Delta^-|^2}}\\
&\quad\times\ln\frac{\sqrt{q_4^2+\xi^2q^2+4|\Delta^-|^2}+\sqrt{q_4^2+\xi^2q^2}}{\sqrt{q_4^2+\xi^2q^2+4|\Delta^-|^2}-\sqrt{q_4^2+\xi^2q^2}}.
\end{aligned}
\tag{B13}
$$

In different limits, we obtain the following behavior:

$$
J^{\mu\nu}_{\Delta}(q)=\begin{cases}
-\frac{\mu^2}{3\pi^2}\left(g^{\mu0}g^{\nu0}+\frac{1}{2}g^{\mu\nu}\right)+\delta J^{(2)}(q), & \text{for}\quad |q_4|,q\ll 2|\Delta^-|;\\
-\frac{2\mu^2|\Delta^-|^2}{3\pi^2q_4^2}\ln\frac{q_4^2}{|\Delta^-|^2}\left(g^{\mu0}g^{\nu0}+\frac{1}{2}g^{\mu\nu}\right), & \text{for}\quad q,|\Delta^-|\ll|q_4|;\\
-\frac{\mu^2|\Delta^-|^2}{4\pi|q_4|q}\ln\frac{4q_4^2}{|\Delta^-|^2}\left(g^{\mu0}g^{\nu0}+g^{\mu\nu}+\frac{\vec{q}^\mu\vec{q}^\nu}{q^2}\right), & \text{for}\quad |\Delta^-|\ll|q_4|\ll q;\\
-\frac{\mu^2|\Delta^-|}{8q}\left(g^{\mu0}g^{\nu0}+g^{\mu\nu}+\frac{\vec{q}^\mu\vec{q}^\nu}{q^2}\right), & \text{for}\quad |q_4|\ll|\Delta^-|\ll q,
\end{cases}
\tag{B14}
$$

where (in Minkowski space)

$$
\delta J^{(2)}(q)=\frac{\mu^2}{90\pi^2|\Delta^-|^2}\left[g^{\mu0}g^{\nu0}(2q^2-5q_0^2)+\frac{1}{2}g^{\mu\nu}(q^2-5q_0^2)-\vec{q}^\mu\vec{q}^\nu\right].
\tag{B15}
$$

As one could see, the imaginary part of $J^{\mu\nu}_{\Delta}(q)$ in Eq. (B12) appears only for $q_0>2|\Delta^-|$. Its explicit form is given in terms of the elliptic integral of the first, $F(\varphi,z)$, and second kind, $E(\varphi,z)$, and the hypergeometric function,

$$
\begin{aligned}
\mathrm{Im}\left[J^{\mu\nu}_{\Delta}(q)\right] &= \frac{\mu^2|\Delta^-|^2}{2\pi}\frac{\theta\left(q_0^2-q^2-4|\Delta^-|^2\right)}{\sqrt{q^2}\sqrt{q_0^2-4|\Delta^-|^2}}\left\{\left(g^{\mu\nu}+g^{\mu0}g^{\nu0}+\frac{\vec{q}^\mu\vec{q}^\nu}{q^2}\right)F\left(\arcsin\sqrt{\frac{q^2}{q_0^2}},\frac{q_0^2}{q_0^2-4|\Delta^-|^2}\right)\right.\\
&\left.-\frac{q_0^2-4|\Delta^-|^2}{q_0^2}\left(g^{\mu\nu}-g^{\mu0}g^{\nu0}+3\frac{\vec{q}^\mu\vec{q}^\nu}{q^2}\right)\left[F\left(\arcsin\sqrt{\frac{q^2}{q_0^2}},\frac{q_0^2}{q_0^2-4|\Delta^-|^2}\right)-E\left(\arcsin\sqrt{\frac{q^2}{q_0^2}},\frac{q_0^2}{q_0^2-4|\Delta^-|^2}\right)\right]\right\}\\
&+\frac{\mu^2|\Delta^-|^2}{2\pi}\frac{\theta\left(4|\Delta^-|^2+q^2-q_0^2\right)\theta\left(q_0^2-4|\Delta^-|^2\right)}{\sqrt{q^2}\sqrt{q_0^2}}\left\{\left(g^{\mu\nu}+g^{\mu0}g^{\nu0}+\frac{\vec{q}^\mu\vec{q}^\nu}{q^2}\right)K\left(\frac{q_0^2-4|\Delta^-|^2}{q_0^2}\right)\right.\\
&\left.-\frac{\pi(q_0^2-4|\Delta^-|^2)}{4q^2}\left(g^{\mu\nu}-g^{\mu0}g^{\nu0}+3\frac{\vec{q}^\mu\vec{q}^\nu}{q^2}\right)\,{}_2F_1\left(\frac{1}{2},\frac{3}{2},2,\frac{q_0^2-4|\Delta^-|^2}{q_0^2}\right)\right\}.
\end{aligned}
\tag{B16}
$$

Similarly, let us calculate the following quantity:

$$
\begin{aligned}
I^{\mu\nu}_{\Delta}(q) &= i\int \frac{d^4p}{(2\pi)^4}\mathrm{tr}_D\left[\gamma^\mu\gamma^0\left(\frac{p_0+\epsilon^-_p}{E^-_\Delta(p)}\Lambda^-_p+\frac{p_0-\epsilon^+_p}{E^+_\Delta(p)}\Lambda^+_p\right)\gamma^\nu\gamma^0\left(\frac{p_0-q_0+\epsilon^-_{p-q}}{E^-_\Delta(p-q)}\Lambda^-_{p-q}+\frac{p_0-q_0-\epsilon^+_{p-q}}{E^+_\Delta(p-q)}\Lambda^+_{p-q}\right)\right]\\
&= i\int \frac{d^4p}{(2\pi)^4}\mathrm{tr}_D\left[\gamma^\mu\gamma^0\left(\frac{p_0+\epsilon^+_p}{E^+_\Delta(p)}\Lambda^-_p+\frac{p_0-\epsilon^-_p}{E^-_\Delta(p)}\Lambda^+_p\right)\gamma^\nu\gamma^0\left(\frac{p_0-q_0+\epsilon^+_{p-q}}{E^+_\Delta(p-q)}\Lambda^-_{p-q}+\frac{p_0-q_0-\epsilon^-_{p-q}}{E^-_\Delta(p-q)}\Lambda^+_{p-q}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
&\simeq \frac{\mu^2}{3\pi^2}\left(g^{\mu\nu}-g^{\mu 0}g^{\nu 0}\right)+2i\int\frac{d^4p}{(2\pi)^4}\left(g^{\mu 0}+\frac{\vec{p}^{\mu}}{|\vec{p}|}\right)\left(g^{\nu 0}+\frac{\vec{p}^{\nu}}{|\vec{p}|}\right)\frac{(p_0+\epsilon_p^-)(p_0-q_0+\epsilon_{p-q}^-)}{E_\Delta^-(p)E_\Delta^-(p-q)}\\
&\simeq \frac{\mu^2}{3\pi^2}\left(g^{\mu\nu}-g^{\mu 0}g^{\nu 0}\right)+2i\int_0^1 dx\int\frac{dp_0d^3p}{(2\pi)^4}\left(g^{\mu 0}+\frac{\vec{p}^{\mu}}{|\vec{p}|}\right)\left(g^{\nu 0}+\frac{\vec{p}^{\nu}}{|\vec{p}|}\right)\\
&\times\frac{\left[p_0+\epsilon_p^-+x(q_0+\xi q)\right]\left[p_0+\epsilon_p^- -(1-x)(q_0+\xi q)\right]}{\left[p_0^2-(\epsilon_p^-)^2-x(1-x)(\xi^2q^2-q_0^2)-|\Delta^-|^2+i\varepsilon\right]^2}.
\end{aligned}
\tag{B17}
$$

So, we derive

$$
\begin{aligned}
I_\Delta^{\mu\nu}(q)&=\frac{\mu^2}{3\pi^2}\left(g^{\mu\nu}-g^{\mu 0}g^{\nu 0}\right)+\frac{\mu^2}{4\pi^2}\int_0^1 dx\int_{-1}^1 d\xi\frac{|\Delta^-|^2+2x(1-x)(q_0q\xi+q^2\xi^2)}{|\Delta^-|^2+x(1-x)(q^2\xi^2-q_0^2)-i\varepsilon}\\
&\times\left[\frac{1}{2}(3-\xi^2)g^{\mu 0}g^{\nu 0}-\frac{1}{2}(1-\xi^2)g^{\mu\nu}-\frac{1}{2}(1-3\xi^2)\frac{\vec{q}^{\mu}\vec{q}^{\nu}}{q^2}+\xi\left(g^{\mu 0}\frac{\vec{q}^{\nu}}{q}+g^{\nu 0}\frac{\vec{q}^{\mu}}{q}\right)\right]\\
&=\frac{\mu^2}{3\pi^2}\left(g^{\mu\nu}-g^{\mu 0}g^{\nu 0}\right)+\frac{\mu^2}{4\pi^2}\int_0^1 dx\int_0^1 d\xi\left[(3-\xi^2)g^{\mu 0}g^{\nu 0}-(1-\xi^2)g^{\mu\nu}-(1-3\xi^2)\frac{\vec{q}^{\mu}\vec{q}^{\nu}}{q^2}\right]\\
&\times\frac{|\Delta^-|^2+2x(1-x)q^2\xi^2}{|\Delta^-|^2+x(1-x)(q^2\xi^2-q_0^2)-i\varepsilon}\\
&+\frac{\mu^2}{\pi^2}\int_0^1 dx\int_0^1 d\xi\left(g^{\mu 0}\frac{\vec{q}^{\nu}}{q}+g^{\nu 0}\frac{\vec{q}^{\mu}}{q}\right)\frac{x(1-x)qq_0\xi^2}{|\Delta^-|^2+x(1-x)(q^2\xi^2-q_0^2)-i\varepsilon}.
\end{aligned}
\tag{B18}
$$

After switching to Eucledian momenta ($q_0=iq_4$), we could also perform the integration over x, and get the following result:

$$
\begin{aligned}
I_\Delta^{\mu\nu}(q)&=I_{HDL}^{\mu\nu}(q)+\frac{\mu^2}{2\pi^2}\int_0^1 d\xi\left[(3-\xi^2)g^{\mu 0}g^{\nu 0}-(1-\xi^2)g^{\mu\nu}-(1-3\xi^2)\frac{\vec{q}^{\mu}\vec{q}^{\nu}}{q^2}\right]\\
&\times\frac{|\Delta^-|^2(q_4^2-\xi^2q^2)}{\sqrt{q_4^2+\xi^2q^2+4|\Delta^-|^2}(q_4^2+\xi^2q^2)^{3/2}}\ln\frac{\sqrt{q_4^2+\xi^2q^2+4|\Delta^-|^2}+\sqrt{q_4^2+\xi^2q^2}}{\sqrt{q_4^2+\xi^2q^2+4|\Delta^-|^2}-\sqrt{q_4^2+\xi^2q^2}}\\
&-\frac{2\mu^2}{\pi^2}\frac{iq_4}{q}\int_0^1 d\xi\frac{\left(g^{\mu 0}\frac{\vec{q}^{\nu}}{q}+g^{\nu 0}\frac{\vec{q}^{\mu}}{q}\right)|\Delta^-|^2\xi^2q^2}{\sqrt{q_4^2+\xi^2q^2+4|\Delta^-|^2}(q_4^2+\xi^2q^2)^{3/2}}\ln\frac{\sqrt{q_4^2+\xi^2q^2+4|\Delta^-|^2}+\sqrt{q_4^2+\xi^2q^2}}{\sqrt{q_4^2+\xi^2q^2+4|\Delta^-|^2}-\sqrt{q_4^2+\xi^2q^2}}.
\end{aligned}
\tag{B19}
$$

The following asymptotes are valid

$$
I_\Delta^{\mu\nu}(q)-I_{HDL}(q)=\begin{cases}
-I_{HDL}(q)+\frac{\mu^2}{3\pi^2}\left(g^{\mu 0}g^{\nu 0}+\frac{1}{2}g^{\mu\nu}\right)+\delta I^{(2)}(q), & \text{for } |q_4|,q\ll 2|\Delta^-|;\\
\frac{\mu^2|\Delta^-|^2}{3\pi^2q_4^2}\ln\frac{q_4^2}{|\Delta^-|^2}\left(4g^{\mu 0}g^{\nu 0}-g^{\mu\nu}+2\frac{g^{\mu 0}\vec{q}^{\nu}+\vec{q}^{\mu}g^{\nu 0}}{iq_4}\right), & \text{for } q,|\Delta^-|\ll|q_4|;\\
-\frac{\mu^2|\Delta^-|^2}{4\pi|q_4|q}\ln\frac{4q_4^2}{|\Delta^-|^2}\left(3g^{\mu 0}g^{\nu 0}-g^{\mu\nu}-\frac{\vec{q}^{\mu}\vec{q}^{\nu}}{q^2}+4iq_4\frac{g^{\mu 0}\vec{q}^{\nu}+\vec{q}^{\mu}g^{\nu 0}}{q^2}\right), & \text{for } |\Delta^-|\ll|q_4|\ll q;\\
-\frac{\mu^2|\Delta^-|}{8q}\left(3g^{\mu 0}g^{\nu 0}-g^{\mu\nu}-\frac{\vec{q}^{\mu}\vec{q}^{\nu}}{q^2}+4iq_4\frac{g^{\mu 0}\vec{q}^{\nu}+\vec{q}^{\mu}g^{\nu 0}}{q^2}\right), & \text{for } |q_4|\ll|\Delta^-|\ll q.
\end{cases}
\tag{B20}
$$

where (in Minkowski space) $\delta I^{(2)}(q)$ reads

$$
\delta I^{(2)}(q)=\frac{\mu^2}{90\pi^2|\Delta^-|^2}\left[g^{\mu 0}g^{\nu 0}(3q^2+10q_0^2)-\frac{1}{2}g^{\mu\nu}(q^2+5q_0^2)+\vec{q}^{\mu}\vec{q}^{\nu}\right]+\frac{\mu^2q_0\left(\vec{q}^{\mu}u^{\nu}+u^{\mu}\vec{q}^{\nu}\right)}{18\pi^2|\Delta^-|^2}.
\tag{B21}
$$

Finally, we calculate the last type of integrals,

$$
\begin{aligned}
\tilde{I}_\Delta^{\mu\nu}(q)&=i\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}_D\left[\gamma^\mu\gamma^0\left(\frac{p_0+\epsilon_p^-}{E_\Delta^-(p)}\Lambda_p^-+\frac{p_0-\epsilon_p^+}{E_\Delta^+(p)}\Lambda_p^+\right)\gamma^\nu\gamma^0\left(\frac{1}{p_0-q_0-\epsilon_{p-q}^-}\Lambda_{p-q}^-+\frac{1}{p_0-q_0+\epsilon_{p-q}^+}\Lambda_{p-q}^+\right)\right]\\
&=i\int\frac{d^4p}{(2\pi)^4}\mathrm{tr}_D\left[\gamma^\mu\gamma^0\left(\frac{p_0+\epsilon_p^+}{E_\Delta^+(p)}\Lambda_p^-+\frac{p_0-\epsilon_p^-}{E_\Delta^-(p)}\Lambda_p^+\right)\gamma^\nu\gamma^0\left(\frac{1}{p_0-q_0-\epsilon_{p-q}^+}\Lambda_{p-q}^-+\frac{1}{p_0-q_0+\epsilon_{p-q}^-}\Lambda_{p-q}^+\right)\right]\\
&\simeq\frac{\mu^2}{3\pi^2}\left(g^{\mu\nu}-g^{\mu 0}g^{\nu 0}\right)+2i\int\frac{d^4p}{(2\pi)^4}\left(g^{\mu 0}+\frac{\vec{p}^{\mu}}{|\vec{p}|}\right)\left(g^{\nu 0}+\frac{\vec{p}^{\nu}}{|\vec{p}|}\right)\frac{(p_0+\epsilon_p^-)(p_0-q_0+\epsilon_{p-q}^-)}{[p_0^2-(\epsilon_p^-)^2-|\Delta^-|^2]E_\Delta^-(p-q)}
\end{aligned}
$$

$$
\begin{aligned}
&\simeq \frac{\mu^2}{3\pi^2}\left(g^{\mu\nu}-g^{\mu 0}g^{\nu 0}\right)+2i\int_0^1 dx\int\frac{dp_0 d^3p}{(2\pi)^4}\left(g^{\mu 0}+\frac{\vec{p}^{\mu}}{|\vec{p}|}\right)\left(g^{\nu 0}+\frac{\vec{p}^{\nu}}{|\vec{p}|}\right)\\
&\times\frac{\left[p_0+\epsilon_p^- + x(q_0+\xi q)\right]\left[p_0+\epsilon_p^- -(1-x)(q_0+\xi q)\right]}{\left[p_0^2-(\epsilon_p^-)^2-x(1-x)(\xi^2q^2-q_0^2)-(1-x)|\Delta^-|^2+i\varepsilon\right]^2}\\
&\simeq \frac{\mu^2}{3\pi^2}\left(g^{\mu\nu}-g^{\mu 0}g^{\nu 0}\right)+\frac{\mu^2}{4\pi^2}\int_0^1 dx\int_0^1 d\xi\frac{\left[(3-\xi^2)g^{\mu 0}g^{\nu 0}-(1-\xi^2)g^{\mu\nu}-(1-3\xi^2)\frac{\vec{q}^{\mu}\vec{q}^{\nu}}{q^2}\right](|\Delta^-|^2+2xq^2\xi^2)}{|\Delta^-|^2+x(q^2\xi^2-q_0^2)-i\varepsilon}\\
&\quad+\frac{\mu^2}{\pi^2}\int_0^1 dx\int_0^1 d\xi\left(g^{\mu 0}\frac{\vec{q}^{\nu}}{q}+g^{\nu 0}\frac{\vec{q}^{\mu}}{q}\right)\frac{xqq_0\xi^2}{|\Delta^-|^2+x(q^2\xi^2-q_0^2)-i\varepsilon}. \qquad \text{(B22)}
\end{aligned}
$$

After switching to Eucledian momenta ($q_0=iq_4$), we could also perform the integration over x, and get the following result:

$$
\begin{aligned}
\tilde{I}_{\Delta}^{\mu\nu}(q)&=I_{HDL}^{\mu\nu}(q)+\frac{\mu^2}{4\pi^2}\int_0^1 d\xi\left[(3-\xi^2)g^{\mu 0}g^{\nu 0}-(1-\xi^2)g^{\mu\nu}-(1-3\xi^2)\frac{\vec{q}^{\mu}\vec{q}^{\nu}}{q^2}\right]\frac{|\Delta^-|^2(q_4^2-\xi^2q^2)}{(q_4^2+\xi^2q^2)^2}\ln\frac{|\Delta^-|^2+q_4^2+\xi^2q^2}{|\Delta^-|^2}\\
&-\frac{\mu^2}{\pi^2}\frac{iq_4}{q}\int_0^1 d\xi\left(g^{\mu 0}\frac{\vec{q}^{\nu}}{q}+g^{\nu 0}\frac{\vec{q}^{\mu}}{q}\right)\frac{|\Delta^-|^2\xi^2q^2}{(q_4^2+\xi^2q^2)^2}\ln\frac{|\Delta^-|^2+q_4^2+\xi^2q^2}{|\Delta^-|^2}. \qquad \text{(B23)}
\end{aligned}
$$

The following asymptotes are valid

$$
\tilde{I}_{\Delta}^{\mu\nu}(q)-I_{HDL}(q)=\begin{cases}
-I_{HDL}(q)+\frac{\mu^2}{3\pi^2}\left(g^{\mu 0}g^{\nu 0}+\frac{1}{2}g^{\mu\nu}\right)+\delta\tilde{I}^{(2)}(q), & \text{for } |q_4|,q\ll 2|\Delta^-|;\\
\frac{\mu^2|\Delta^-|^2}{6\pi^2q_4^2}\ln\frac{q_4^2}{|\Delta^-|^2}\left(4g^{\mu 0}g^{\nu 0}-g^{\mu\nu}+2\frac{g^{\mu 0}\vec{q}^{\nu}+\vec{q}^{\mu}g^{\nu 0}}{iq_4}\right), & \text{for } q,|\Delta^-|\ll|q_4|;\\
-\frac{\mu^2|\Delta^-|^2}{8\pi|q_4|q}\left(3g^{\mu 0}g^{\nu 0}-g^{\mu\nu}-\frac{\vec{q}^{\mu}\vec{q}^{\nu}}{q^2}+iq_4\frac{g^{\mu 0}\vec{q}^{\nu}+\vec{q}^{\mu}g^{\nu 0}}{q^2}\right), & \text{for } |\Delta^-|\ll|q_4|\ll q;\\
-\frac{\mu^2|\Delta^-|}{4\pi q}\left(3g^{\mu 0}g^{\nu 0}-g^{\mu\nu}-\frac{\vec{q}^{\mu}\vec{q}^{\nu}}{q^2}+iq_4\frac{g^{\mu 0}\vec{q}^{\nu}+\vec{q}^{\mu}g^{\nu 0}}{q^2}\right), & \text{for } |q_4|\ll|\Delta^-|\ll q.
\end{cases} \qquad \text{(B24)}
$$

where (in Minkowski space)

$$
\delta\tilde{I}^{(2)}(q)=\frac{\mu^2}{30\pi^2|\Delta^-|^2}\left[g^{\mu 0}g^{\nu 0}(3q^2+10q_0^2)-\frac{1}{2}g^{\mu\nu}(q^2+5q_0^2)+\vec{q}^{\mu}\vec{q}^{\nu}\right]+\frac{\mu^2q_0\left(\vec{q}^{\mu}g^{\nu 0}+g^{\mu 0}\vec{q}^{\nu}\right)}{5\pi^2|\Delta^-|^2}. \qquad \text{(B25)}
$$

APPENDIX C: GAUGE INVARIANCE AND POLARIZATION TENSOR

By making use of the definitions in Eq. (B8) as well as the general structure of the tensors $J_{\Delta}^{\mu\nu}(q)$, $I_{\Delta}^{\mu\nu}(q)$ and $\tilde{I}_{\Delta}^{\mu\nu}(q)$, given in the preceding Appendix, it is easy to show that all three polarization tensors $\Pi_i^{\mu\nu}(q)$ ($i=1,4,8$) take the following general form:

$$
\Pi_i^{\mu\nu}(q)=\Pi_i^{(1)}(q)O^{(1)\mu\nu}(q)+\Pi_i^{(2)}(q)O^{(2)\mu\nu}(q)+\Pi_i^{(3)}(q)O^{(3)\mu\nu}(q)+\Pi_i^{(4)}(q)O^{(4)\mu\nu}(q). \qquad \text{(C1)}
$$

In this representation, we use the following set of four tensors [23]:

$$
O_{\mu\nu}^{(1)}(q)=g_{\mu\nu}-u_\mu u_\nu+\frac{\vec{q}_\mu\vec{q}_\nu}{|\vec{q}|^2}, \qquad \text{(C2a)}
$$

$$
O_{\mu\nu}^{(2)}(q)=u_\mu u_\nu-\frac{\vec{q}_\mu\vec{q}_\nu}{|\vec{q}|^2}-\frac{q_\mu q_\nu}{q^2}, \qquad \text{(C2b)}
$$

$$
O_{\mu\nu}^{(3)}(q)=\frac{q_\mu q_\nu}{q^2}, \qquad \text{(C2c)}
$$

$$
O_{\mu\nu}^{(4)}(q)=O_{\mu\lambda}^{(2)}u^\lambda\frac{q_\nu}{|\vec{q}|}+\frac{q_\mu}{|\vec{q}|}u^\lambda O_{\lambda\nu}^{(2)}. \qquad \text{(C2d)}
$$

The first three of them are the same projectors of the magnetic, electric and unphysical (longitudinal in a 3+1 dimensional sense) modes of gluons which were used in Ref. [7]. In addition, here we also introduced the intervening operator $O_{\mu\nu}^{(4)}(q)$, mixing the electric and unphysical modes [23]. Note that $u_\mu=(1,0,0,0)$ and $\vec{q}_\mu=q_\mu-(u\cdot q)u_\mu$.

By making use of the representations for different types of the polarization tensors in Appendix B, we derive the following explicit infrared ($|q_0|, q \ll |\Delta^-|$) asymptotes:

$$\Pi_1^{\mu\nu}(q) = \frac{2\alpha_s \mu^2}{9\pi|\Delta^-|^2}\left[g^{\mu 0} g^{\nu 0}(q^2+q_0^2) - q_0^2 g^{\mu\nu} + q_0\left(\vec{q}^{\mu} g^{\nu 0} + g^{\mu 0}\vec{q}^{\nu}\right)\right]$$
$$= -\frac{2\alpha_s \mu^2}{9\pi|\Delta^-|^2}\left[q_0^2 O^{(1)\mu\nu}(q) + (q_0^2 - q^2) O^{(2)\mu\nu}(q)\right], \tag{C3}$$

$$\Pi_4^{\mu\nu}(q) = \frac{3}{2}\Pi_8^{\mu\nu}(q) = \frac{2\alpha_s\mu^2}{3\pi}\left[O^{(1)\mu\nu}(q) + \frac{q_0^2 - 3q^2}{q_0^2 - q^2} O^{(2)\mu\nu}(q) + \frac{3q_0^2 - q^2}{q_0^2 - q^2} O^{(3)\mu\nu}(q) + \frac{2q_0 q}{q_0^2 - q^2} O^{(4)\mu\nu}(q)\right]. \tag{C4}$$

Similarly, we derive the asymptotes in other regions of interest:

$$\Pi_1^{\mu\nu}(q) - 4\pi\alpha_s I_{HDL}^{\mu\nu}(q) = \begin{cases} -\frac{8\alpha_s\mu^2|\Delta^-|^2}{3\pi q_4^2}\ln\frac{q_4^2}{|\Delta^-|^2}\left[O^{(1)}(q) + O^{(2)}(q)\right], & \text{for } q, |\Delta^-| \ll |q_4|; \\ -\frac{\alpha_s\mu^2|\Delta^-|^2}{|q_4| q}\ln\frac{4q_4^2}{|\Delta^-|^2} O^{(1)}(q), & \text{for } |\Delta^-| \ll |q_4| \ll q; \\ -\frac{2\pi\alpha_s\mu^2|\Delta^-|}{q} O^{(2)}(q), & \text{for } |q_4| \ll |\Delta^-| \ll q, \end{cases} \tag{C5}$$

$$\Pi_4^{\mu\nu}(q) - 4\pi\alpha_s I_{HDL}^{\mu\nu}(q) = \begin{cases} -\frac{2\alpha_s\mu^2|\Delta^-|^2}{3\pi q_4^2}\ln\frac{q_4^2}{|\Delta^-|^2}\left[O^{(1)}(q) + O^{(2)}(q) - 3O^{(3)}(q) + \frac{2q}{iq_4}O^{(4)}(q)\right], & \text{for } q, |\Delta^-| \ll |q_4|; \\ \frac{\alpha_s\mu^2|\Delta^-|^2}{2|q_4| q}\left[O^{(1)}(q) - O^{(2)}(q) + \frac{iq_4}{q}O^{(4)}(q)\right], & \text{for } |\Delta^-| \ll |q_4| \ll q; \\ \frac{\pi\alpha_s\mu^2|\Delta^-|}{q}\left[O^{(1)}(q) - O^{(2)}(q) + \frac{iq_4}{q}O^{(4)}(q)\right], & \text{for } |q_4| \ll |\Delta^-| \ll q, \end{cases} \tag{C6}$$

and

$$\Pi_8^{\mu\nu}(q) - 4\pi\alpha_s I_{HDL}^{\mu\nu}(q) = \begin{cases} \frac{8\alpha_s\mu^2|\Delta^-|^2}{3\pi q_4^2}\ln\frac{q_4^2}{|\Delta^-|^2}\left[O^{(3)}(q) + \frac{2q}{3iq_4}O^{(4)}(q)\right], & \text{for } q, |\Delta^-| \ll |q_4|; \\ \frac{2\alpha_s\mu^2|\Delta^-|^2}{3|q_4| q}\ln\frac{4q_4^2}{|\Delta^-|^2}\left[O^{(1)}(q) + \frac{4iq_4}{q}O^{(4)}(q)\right], & \text{for } |\Delta^-| \ll |q_4| \ll q; \\ \frac{\pi\alpha_s\mu^2|\Delta^-|}{3q}\left[O^{(1)}(q) + \frac{4iq_4}{q}O^{(4)}(q)\right], & \text{for } |q_4| \ll |\Delta^-| \ll q. \end{cases} \tag{C7}$$

As is easy to check, $\Pi_1^{\mu\nu}(q)$ is transverse everywhere, while $\Pi_4^{\mu\nu}(q)$ and $\Pi_8^{\mu\nu}(q)$ are not. From the structure of the polarization tensor $\Pi_1^{\mu\nu}(q)$ in Eq. (C3), we derive the explicit expression for the dielectric constant in the far infrared region where the three gluons of the unbroken $SU(2)_c$ decouple from the other degrees of freedom. It coincides with the result in Ref. [19] which is quoted in Eq. (31). We also see that the magnetic permeability is equal 1, because there is no magnetic contribution $q^2 O^{(1)\mu\nu}(q)$ on the right hand side of Eq. (C3) [notice that $q_0^2 O^{(1)\mu\nu}(q)$ is the electric type contribution]. Let us also notice that the general expression for $\Pi_1^{\mu\nu}(q)$ is transverse for all momenta. To see this, we derive the following representation:

$$\Pi_1^{\mu\nu}(q) = 4\pi\alpha_s I_{HDL}^{\mu\nu}(q) - \frac{4\alpha_s\mu^2|\Delta^-|^2}{\pi}\int_0^1 \frac{d\xi\left[q_4^2(1-\xi^2)O^{(1)\mu\nu}(q) + 2\xi^2(q_4^2+q^2)O^{(2)\mu\nu}(q)\right]}{\sqrt{q_4^2+\xi^2 q^2 + 4|\Delta^-|^2}(q_4^2+\xi^2q^2)^{3/2}}$$
$$\times \ln\frac{\sqrt{q_4^2+\xi^2q^2+4|\Delta^-|^2} + \sqrt{q_4^2+\xi^2q^2}}{\sqrt{q_4^2+\xi^2q^2+4|\Delta^-|^2} - \sqrt{q_4^2+\xi^2q^2}}. \tag{C8}$$

From the asymptotes in Eqs. (B14) and (B20), one might get the impression that the transversality of $\Pi_1^{\mu\nu}(q) \equiv 4\pi\alpha_s[J^{\mu\nu}(q) + I^{\mu\nu}(q)]$ is not exact. This is just an artifact of using the expansion for either the limit of $|q_4| \ll q$ or $q \ll |q_4|$. To test the condition $q_\mu \Pi_1^{\mu\nu}(q) = 0$, one has to keep the subleading corrections to such expansions.

It is straightforward to check that the general expression in Eq. (24) becomes strongly gauge dependent if the one-loop polarization tensors $\Pi_4^{\mu\nu}(q)$ and $\Pi_8^{\mu\nu}(q)$ with nonzero longitudinal components are used in the calculation. Moreover, the corresponding gauge dependent contribution to the value of the mass comes from the region of small momenta, $|q_4|, q \ll |\Delta^-|$, and it is logarithmically divergent,

$$M_\lambda^2 \sim \lambda^2\alpha_s^2\mu^2 \ln\frac{|\Delta^-|}{\epsilon_{IR}}, \tag{C9}$$

where λ is the gauge fixing parameter and ϵ_{IR} is the infrared cutoff. In deriving this result, we used the asymptotes presented in Eq. (C4). This result clearly demonstrates that the presence of the longitudinal components in the

polarization tensors $\Pi_4^{\mu\nu}(q)$ and $\Pi_8^{\mu\nu}(q)$ is not acceptable. In the complete theory these components should be zero, because the Ward (Slavnov-Taylor) identities imply that the polarization tensor should be transverse. As we mentioned in the main text, it is the contributions of the (would be) NG bosons which should be added to the polarization tensors in order to restore the transversality.

As it was already pointed out in Sec. IV, one has to use the non-linear realization of color symmetry breaking for that purpose. The polarization tensor can be found from the corresponding low energy effective action (a similar approach for improving the polarization tensor has been recently considered in Ref. [23]). The effective action for the two flavor dense QCD was described in Ref. [24]. It is given in terms of the Maurer-Cartan one-form,

$$\omega_\mu = \mathcal{V}^\dagger i(\partial_\mu - ig\hat{A}_\mu)\mathcal{V}, \tag{C10}$$

where the field $\mathcal{V}$ parametrizes the coset space $SU(3)_c \times U(1)_B/SU(2)\times\tilde{U}(1)_B$. Its explicit form reads

$$\mathcal{V} = \exp\left[i\sum_{A=4}^{7}\phi^A T^A + i\frac{\phi^8}{3}\left(I+\sqrt{3}T^8\right)\right]. \tag{C11}$$

In the last expression, we took into account that one of the broken generators is $\left(I+\sqrt{3}T^8\right)/3$ rather than T^8. The fields ϕ^A $(A=4,\ldots,8)$ describe dynamical (would be) Nambu-Goldstone degrees of freedom which should necessarily appear in the low-energy description of the color superconducting phase of dense QCD (unless a unitary gauge is used). The interaction of gluons and would be NG bosons with fermions is described by the following term [24]:

$$L_f = \bar{\psi}(i\partial\!\!\!/ + \mu\gamma^0 + \gamma^\mu\omega_\mu)\psi. \tag{C12}$$

Therefore, after integrating out fermions and high momenta gluons (with $q \gg |q_4| \gg |\Delta^-|$) which are mostly responsible for generating the gap [7], the one-loop approximation of the effective action is mimicked by the following expression:

$$S_{eff} = -\frac{i}{2}\mathrm{Tr}\ln\left(\begin{array}{cc}\mathcal{K}_+ & \Delta\\ \tilde{\Delta} & \mathcal{K}_-\end{array}\right) = -\frac{i}{2}\mathrm{Tr}\ln\left[\mathcal{K}_-\mathcal{K}_+\right] - \frac{i}{4}\mathrm{Tr}\ln\left[1-\mathcal{K}_+^{-1}\Delta\mathcal{K}_-^{-1}\tilde{\Delta}\right], \tag{C13}$$

where

$$\mathcal{K}_+ \equiv i\partial\!\!\!/ + \mu\gamma^0 + \mathcal{V}^\dagger i(\partial\!\!\!/ - ig\hat{A\!\!\!/})\mathcal{V}, \tag{C14}$$

$$\mathcal{K}_- \equiv i\partial\!\!\!/ - \mu\gamma^0 + \mathcal{V}^T i(\partial\!\!\!/ + ig\hat{A\!\!\!/}^T)\mathcal{V}^*. \tag{C15}$$

The quadratic part of the induced effective action, as follows from Eq. (C13), reads

$$\begin{aligned} S_{eff}^{(2)} \simeq -\frac{i}{4}\int\frac{d^4q d^4p}{(2\pi)^8}\mathrm{tr}\Big[&R_{11}(p)f_+(q)R_{11}(p-q)f_+(-q) + R_{12}(p)f_-(q)R_{21}(p-q)f_+(-q)\\ &+ R_{21}(p)f_+(q)R_{12}(p-q)f_-(-q) + R_{22}(p)f_-(q)R_{22}(p-q)f_-(-q)\Big], \end{aligned} \tag{C16}$$

where, by derivation,

$$f_+(q) = \gamma^\mu\left[gA_\mu^A(q)T^A + iq_\mu\sum_{A=4}^{7}\phi^A(q)T^A + iq_\mu\frac{\phi^8(q)}{3}\left(I+\sqrt{3}T^8\right)\right], \tag{C17}$$

$$f_-(q) = \gamma^\mu\left[-gA_\mu^A(q)(T^A)^T - iq_\mu\sum_{A=4}^{7}\phi^A(q)(T^A)^T - iq_\mu\frac{\phi^8(q)}{3}\left(I+\sqrt{3}T^8\right)\right]. \tag{C18}$$

(here we expanded $\mathcal{V}$ in powers of ϕ^A). By substituting the last expressions into Eq. (C16), we arrive at the result in the following form:

$$\begin{aligned} S_{eff}^{(2)} \simeq \frac{1}{2}\int\frac{d^4q}{(2\pi)^4}\Bigg\{&\sum_{A=1}^{3}A_\mu^A(-q)\Pi_1^{\mu\nu}(q)A_\nu^A(q)\\ &+\sum_{A=4}^{7}\left[A_\mu^A(-q)-\frac{i}{g}q_\mu\phi^A(-q)\right]\Pi_4^{\mu\nu}(q)\left[A_\nu^A(q)+\frac{i}{g}q_\nu\phi^A(q)\right]\\ &+\left[A_\mu^8(-q)-\frac{i\sqrt{3}}{g}q_\mu\phi^8(-q)\right]\tilde{\Pi}_8^{\mu\nu}(q)\left[A_\nu^8(q)+\frac{i\sqrt{3}}{g}q_\nu\phi^8(q)\right]\Bigg\}, \end{aligned} \tag{C19}$$

where, by derivation,

$$\Pi_8^{\mu\nu}(q) \equiv \frac{1}{3}\left[\tilde{\Pi}_8^{\mu\nu}(q) + 8\pi\alpha_s I_{HDL}^{\mu\nu}(q)\right]. \tag{C20}$$

By integrating out the ϕ^A fields, we arrive at the effective action of the gluon field [compare with the effective action in Ref. [23]]:

$$\begin{aligned} S_{gl}^{(2)} \simeq \frac{1}{2}\int \frac{d^4q}{(2\pi)^4}\Bigg\{ &\sum_{A=1}^{3} A_\mu^A(-q)\Pi_1^{\mu\nu}(q)A_\nu^A(q) \\ &+ \sum_{A=4}^{7} A_{\mu\perp}^A(-q)\left[\Pi_4^{\mu\nu}(q) - \Pi_4^{\mu\mu'}(q)q_{\mu'}\left(q_\lambda \Pi_4^{\lambda\kappa}q_\kappa\right)^{-1} q_{\nu'}\Pi_4^{\nu'\nu}(q)\right]A_{\nu\perp}^A(q) \\ &+ A_{\mu\perp}^8(-q)\left[\tilde{\Pi}_8^{\mu\nu}(q) - \tilde{\Pi}_8^{\mu\mu'}(q)q_{\mu'}\left(q_\lambda \tilde{\Pi}_8^{\lambda\kappa}q_\kappa\right)^{-1} q_{\nu'}\tilde{\Pi}_8^{\nu'\nu}(q)\right]A_{\nu\perp}^8(q). \end{aligned} \tag{C21}$$

By making use of the properties of the polarization tensor, we check that this quadratic part of the action splits into three decoupled pieces with different types of gluons. One of them contains the first three gluon that do not feel the Meissner effect. The improved polarization tensors for the five gluons corresponding to broken generators, explicitly transverse, and thus, they can be written in the standard form:

$$\Pi_{i,new}^{\mu\nu}(q) = -\Pi_{i,t}(q)O^{(1)\mu\nu}(q) - \frac{q_0^2 - q^2}{q^2}\Pi_{i,l}(q)O^{(2)\mu\nu}(q), \tag{C22}$$

where

$$\Pi_{i,t}(q) = -\Pi_i^{(1)}(q), \tag{C23}$$

$$\Pi_{i,l}(q) = -\frac{q^2}{q_0^2 - q^2}\left[\Pi_i^{(2)}(q) + \frac{[\Pi_i^{(4)}(q)]^2}{\Pi_i^{(3)}(q)}\right], \tag{C24}$$

with $\Pi_i^{(x)}(q)$ ($i = 4, 8$ and $x = 1, 2, 3, 4$) defined in Eq. (C1).

In the infrared region $|q_4|, q \ll |\Delta^-|$, the improved gluon tensors that correspond to the broken generators read

$$\Pi_{4,new}^{\mu\nu}(q) = \frac{3}{2}\Pi_{8,new}^{\mu\nu}(q) = \frac{2\alpha_s\mu^2}{3\pi}\left[O^{(1)\mu\nu}(q) + \frac{q_0^2 - q^2}{q_0^2 - \frac{1}{3}q^2}O^{(2)\mu\nu}(q)\right]. \tag{C25}$$

Similarly, we get the expressions in other limits:

$$\Pi_{4,new}^{\mu\nu}(q) - 4\pi\alpha_s I_{HDL}^{\mu\nu}(q) = \begin{cases} -\frac{2\alpha_s\mu^2|\Delta^-|^2}{3\pi q_4^2}\ln\frac{q_4^2}{|\Delta^-|^2}\left[O^{(1)}(q) + O^{(2)}(q)\right], & \text{for } q, |\Delta^-| \ll |q_4|; \\ \frac{\alpha_s\mu^2|\Delta^-|^2}{2|q_4|q}\left[O^{(1)}(q) - O^{(2)}(q)\right], & \text{for } |\Delta^-| \ll |q_4| \ll q; \\ \frac{\pi\alpha_s\mu^2|\Delta^-|}{q}\left[O^{(1)}(q) - O^{(2)}(q)\right], & \text{for } |q_4| \ll |\Delta^-| \ll q, \end{cases} \tag{C26}$$

and

$$\Pi_{8,new}^{\mu\nu}(q) - 4\pi\alpha_s I_{HDL}^{\mu\nu}(q) = \begin{cases} 0, & \text{for } q, |\Delta^-| \ll |q_4|; \\ \frac{2\alpha_s\mu^2|\Delta^-|^2}{3|q_4|q}\ln\frac{4q_4^2}{|\Delta^-|^2}O^{(1)}(q), & \text{for } |\Delta^-| \ll |q_4| \ll q; \\ \frac{\pi\alpha_s\mu^2|\Delta^-|}{3q}O^{(1)}(q), & \text{for } |q_4| \ll |\Delta^-| \ll q. \end{cases} \tag{C27}$$

One should notice that the magnetic components [determined by the terms with the projection operator $O^{(1)}(q)$] of the new polarization tensors, are exactly the same as in the one-loop approximation given at the beginning of this Appendix. The electric components are different in general, but they appear to be also essentially the same to the leading order approximation. Most importantly, however, the longitudinal contributions in $\Pi_4^{\mu\nu}(q)$ and $\Pi_8^{\mu\nu}(q)$ are gone now. The immediate consequence of this is that the vacuum energy given by Eq. (24), as well as the masses of the pseudo-NG bosons, become explicitly gauge invariant.

For completeness of presentation, we also note that the contribution of the modified electric components of the polarization tensors to the value of the pseudo-NG boson mass is of order

$$\delta_{el} M^2 \sim \frac{|\Delta^-|^3}{\mu}. \tag{C28}$$

Because of the chemical potential in the denominator, this is clearly a subleading correction as compared to the result in Eq. (28).

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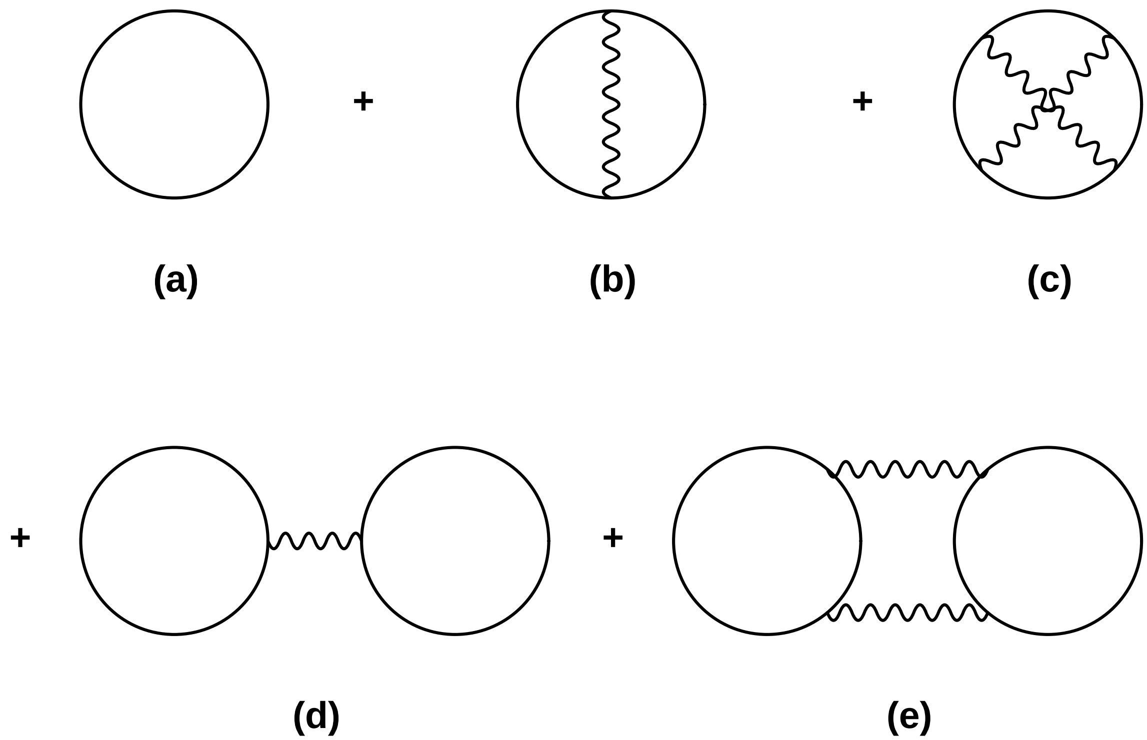

FIG. 1. Vacuum energy diagrams.